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Parent's experience in remote learning during COVID-19 with digital and physical mathematical modelling

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Abstract

In reaction to the COVID-19 pandemic, the government of Luxembourg suspended in-school teaching and learning towards remote teaching. A survey conducted by the Ministry of Education after three weeks of confinement, showed that more than half of the parents faced difficulties when using remote teaching with their students. To tackle this new challenge, we adapted our research to the use of augmented reality, digital and physical mathematical modelling in remote mathematics education for elementary schools. The elementary school students (aged 5 to 12) created cultural artifacts (i.e., Easter egg cups) during the confinement. In this paper, we will describe mathematical modelling in remote teaching and further concentrate on parents' perspectives, who played an essential role in assisting their children. Moreover, we will discuss different didactical principles that emerged from the task design during the study through parents' eyes. Thus, understanding parents' perspectives became highly important in enabling us to improve task designs and related pedagogical approaches in remote teaching. The data collected in this study included semi-structured interviews with students, parents, and teachers as well as questionnaires and field notes. We followed an exploratory stance with our data analyses, primarily utilizing grounded theory (Corbin & Strauss, 1990, 2014) approaches. Through the insights we gained from our findings, we aim to explain how the parents perceived teaching and learning mathematical modelling in our experiments, how they scaffolded the given tasks, and what support they required and would need in future remote teaching.

Keywords: Mathematical modelling, Geometry, Parents, Remote teaching, Educational technology

Introduction

COVID-19 and the confinement induced a series of profound changes to teaching and learning mathematics in elementary schools in Luxembourg. One major change was that schools were closed, and teaching was moved from in-school to complete remote teaching. Teachers, parents, and students needed to identify and adapt to a rather unknown teaching and learning setting. Teachers needed to rethink their methodologies and use a variety of technologies to create a new learning environment for their students. School administrations and specialized services from the Ministry of Education in Luxembourg did however guide teachers in this crucial period. But how were parents perceiving this major change? Although parents took part in the learning and teaching processes before the pandemic by doing homework reviews or test preparations with their children, this meant for them to interfere in a different way. Before the confinement, parents were mainly guided by the class teacher or got feedback in class almost the next day. Hence, in remote teaching, parents needed to anticipate far more often the teaching guidance and organize the teaching with their children. During the pandemic, based on a survey led by the Ministry of Education in Luxembourg, half of the parents reported a difficult start with remote teaching. They stated not having enough time to deal with remote teaching, having a lack of knowledge and skills, not being equipped sufficiently with technologies, or not having all the needed information for students' learning. This raised questions about the teaching that happened during remote teaching. How did these concerns from parents influence mathematical task solving that students received from their teachers? How could parents be best supported? The period of remote teaching in elementary school did last more than eight weeks in a row during the last two trimesters. A long period where students would have learned and experimented with their teachers' topics in mathematics class, such as mathematical modelling with geometric shapes. Mathematical modelling is a central topic in mathematics education in elementary and it is learned through various hands-on learning activities (e.g., apply geometric shapes to real-world objects, combine objects to create shapes), which connect classroom learning to the student's everyday life (Haas et al., 2020; Lavicza et al., 2020). Although, even in in-school settings, research showed that students encounter difficulties in mathematical modelling, while connecting real-life three-dimensional worlds and two-dimensional space (González, 2015). Further, to get a better understanding of geometry and mathematical modelling, students should work their visual-spatial memories (Szucs et al., 2013). Thus, in regular class, they would have used different scaffoldings and materials with the parents, which were not necessarily present in remote teaching. Based on our experiences, we had to pose the question: Do parents in remote teaching have the materials and skills to support their children in getting a better visualization and modelling geometric shapes? Yet, teaching and learning in the digital age are not limited to the classroom and due to the accessibility of educational technology, the interactions on learning content, such as mathematical modelling with geometric shapes, are likely to happen as well at home. Students can use these technologies during class and at home for their learning purposes. Dynamic geometry software on mobile devices, such as GeoGebra 3D Graphing Calculator (Hohenwarter et al., 2020) and Autodesk® Tinkercad® makes it even possible to realize mathematical modelling in nearly any environment. Hence, there are possibilities to support students with materials in experimenting and visualizing geometric shapes in remote teaching. Among these educational technologies software, you find the rather innovative use of augmented reality (AR), as well as digital and the physical mathematical modelling with 3D printing. These technologies are already used in class, with their teachers and peers. However, tasks were scaffolded and organized by a professional of the education system. What would this mean for parents in remote teaching? Even with the materials to teach geometric shapes to their children in remote teaching, would they be able to take the role of the professional and what would they need to do so? What influences would this have on the students' learning? With the start of the confinement on 16 March 2020, the teaching and learning changed to complete remote teaching. We used this opportunity to experiment mathematical modelling study in remote teaching. In this study, parents were able to enroll voluntarily or had been asked by their children's class teacher to participate in an online mathematical modelling challenge and create a cultural artifact (i.e., Easter egg cup). The mathematical modelling tasks were done with complete remote teaching with 44 students, aged from 5 to 13 years. We were able to collect data on the motivation students and their parents perceived during the modelling task, description of how the learning and teaching setting was realized and on the support the parents asked for. Further, we will present how parent's scaffolding could increase or decrease the complexity of a mathematical modelling task with real-world information.

Although we collected data from students, teachers and parents, our main focus in this paper is on the parent's perspectives in remote teaching, as this could be relevant for any further research on parent's participation in remote teaching in mathematical modelling and parent's involvement in teaching in general.

Theoretical framework

Mathematical modelling framework

Mathematical modelling is an element of the main process skills (i.e., problem-solving, reasoning and proof, representation, and modelling) which are taught, according to the national curriculum throughout elementary school in mathematics primary education. Furthermore, according to the curriculum and research, mathematical modelling should connect to real-world situations, problems or objects in the students' living environment (Liljedahl et al., 2016; Singer et al., 2013). However, process skills and mathematical modelling are described slightly different in various research and organisations. Thus, we compared international standards (NCTM, 1999), national standard (MENFP, 2011) and theoretical references mostly used in Luxembourgish schools (Selter & Zannetin, 2019), to have a holistic understanding for our remote teaching and establish a research framework. Apart from language differences (i.e., NCTM in English, national curriculum in French and theoretical references in German), we conclude that mathematical modelling refers, according to the different references, to the application and modelling of mathematical concepts to real-world objects, situations, and problems, where students deploy mathematical skills and knowledge to act on given real-world information (Haas et al., 2020).

Referring to the model of Blum and Leiß (2007), which showed to cover the different references as well, mathematical modelling with real-world information can be described through seven steps. Thus, (1) students construct a situation model based on real-world information and (2) simplify this model to a real problem (e.g., What is the given problem? What do we need to solve?). (3) This real problem is then mathematized (i.e., connecting mathematical knowledge and skills and applying the most appropriate mathematical concept) and (4) worked mathematically. (5) Students interpret the results, (6) proceed to a validation by transferring the mathematical results to real-world and (7) finally expose the results to real-world situation. These steps can be seen as subskills in mathematical modelling and should therefore be considered in development and scaffolding. Mathematical modelling on geometric shapes remained one core topic domain during the confinement in schools. In regular class, teachers would work with students from early childhood to grade 6 on properties, areas, volumes, or combinations. Within a continuous development of process and content skills, teachers would support students through different scaffoldings and feedback. However, in the imposed remote teaching, the regular support in learning mathematical modelling teaching was not possible. Parents were mainly involved, and the professional scaffolding of a teacher was in general not feasible. Therefore, we investigated the use of different technologies to present scaffoldings and support to students in remote teaching.

Augmented reality, digital and physical modelling

We carried out literature reviews on the use, in elementary school, on augmented reality (Bacca Acosta et al., 2019; Billinghurst & Duenser, 2012; Cai et al., 2019; González, 2015; Liu et al., 2019), as well as digital and physical mathematical modelling (Berdik, 2017; Haas et al., 2020; Lavicza et al., 2020; Lieban, 2019; Ng & Chan, 2019). Based on these literature reviews, we focused on four basic principles of Dienes's theory of mathematics (Dienes, 1960) to elaborate the tasks on mathematical modelling in remote teaching. Thus, tasks could reach from reproducing and recreation, to free construction and combination of different geometric shapes. In the remote teaching, we presented tasks which were open to different solution paths. Thus, depending on the scaffolding or the performances in mathematical modelling, produced during the tasks, elementary students experienced tasks differently. With augmented reality elementary school students could have simply transferred the real-world information into three-dimensional digital modelling (i.e., Dynamic Principle) or do a slight modelling of the existing real-world information (i.e., Perceptual Variability Principle), by adding small add-ons. Moreover, students could combine different real-world information (i.e., Mathematical Variability Principle) to a new geometric model or create an entirely new construction (i.e., Constructivity Principle). Referring to the remote teaching challenge we undertook, some students reproduced Easter egg cups, which were almost identical to standard egg cups (i.e., two symmetrical opposed hemispheres) or created complex egg cups with patterns, different modelled geometric shapes and transformed real-world objects. Mathematical modelling with GeoGebra 3D Graphing Calculator or Autodesk® Tinkercad® could support students during the remote teaching and increase the learning outcome in modelling geometric shapes. Thus, the important spatial reasoning in geometry (Sinclair & Bruce, 2015) could be widely supported by these technologies (Clements & Sarama, 2011; Liu et al., 2019). The identification of the needed scaffolding, on how to use the augmented information (Bacca Acosta et al., 2019) plays an important role.

Scaffoldings and task design in mathematical modelling with technologies

Before starting the online remote teaching challenge, we experimented with task design within two grade 4 (age 10 to 13) classes. We wanted to identify which task setting would be more likely to fit into a remote teaching and be adapted to the teaching of the parents. Both classes had the same design to fulfill by using AR, digital and physical mathematical modelling, but in different settings, feedbacks, and scaffoldings. In the first class (Figure 1), students randomly chose three shapes out of a basket and integrated these shapes into their designs. Before starting to work on digital modelling, they draw a design with paper and pencil. After this, they designed an Easter egg cup and verified the functionality of the design with AR.

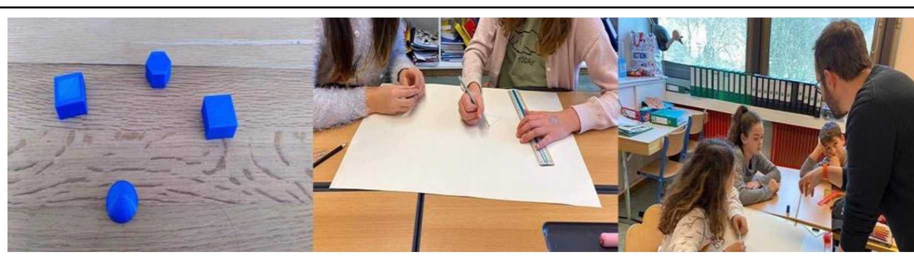

Fig. 1 Tasks realized in the first class [a]

Fig. 2 Tasks realized in the second class

In the second class (Figure 2), the students were free to use the shapes they wanted to integrate in their designs and did not need to create a paper-pencil design. They designed an Egg cup and then verified their functionality of the design with AR.

Thus, the two settings provided us with interesting findings on scaffoldings and supports we should provide in the remote teaching challenge. In the first-class setting, the scaffolding was mostly guided by the teacher and there was less space for students to express their creativity. Students' motivation was high for the interaction with the digital modelling, but low with the paper-pencil design and they felt a hindrance in using the given shapes for their designs. The discussions among students were less focused on their planning and solution processes in mathematical modelling. Students stayed very close to real-world information and the process of mathematization was referring mostly to common geometric concepts.

Thus, to increase the task complexity and the modelling outcome, continuous scaffolding is needed. With this setting, in remote teaching, parents would need to follow a given methodology and apply a large amount of scaffolding, which is done in in-school teaching by a professional teacher. Parents would need a very guided description on the different scaffoldings and some of these scaffolding techniques require skills that are part of the professional training of teachers. In consequence, depending on the parents' educational status, the settings could or could not be done with the required scaffoldings.

In the second class setting, students seemed to be engaged in a more creative and motivated process. Students argued and communicated on their plans and solutions for their designs. They seemed to be in a flow where the task was appropriated, the goal clear, and accepted by the students (Nakamura & Csikszentmihalyi, 2014). Hence in this setting, parents could get involved and interact in the teaching based on their experiences and skills. This situation we would qualify as less scholarly and probably more accessible to a broader public, i.e., parents. However, both settings could be used in the remote teaching as some parents could gain more safety in using a clear scaffolded teaching setting than in a setting that is less controlled and more open to the creativity of the students. Yet, this would require stronger guidance by the parents and could be more difficult to execute for a non-professional teaching person.

Remote teaching challenge

The government in Luxembourg decided to suspend onsite teaching and learning, due to the COVID-19 Pandemic, from 16 March on. Thus, jointly with the Ministry of Education in Luxembourg, we set up a remote teaching challenge by using AR, digital and physical mathematical modelling, building on our previous experiences in class. The task consisted in creating a cultural artifact (i.e., Easter egg cup) in an online challenge. We made the decision to propose this task, as it fitted to the cultural context of Easter, where in regular school students would perform creative Easter activities. We further expected a broader acceptance among the school community with this thematic choice. Students were encouraged to design an Easter egg cup with digital mathematical modelling in GeoGebra 3D Graphing Calculator or Autodesk® Tinkercad® and send their design to an email address we communicated to the parents. To support parents and the students we offered several tutorial videos to explain functions on the digital mathematical modelling software and a dedicated space on the web page. Moreover, parents could reach out by mail or Microsoft Teams (for which each student has a free license) for additional support. Students joined voluntarily or had been enrolled by their class teacher. We had 44 students from ages 5 to 13 who participated in the challenge.

The execution of the tasks was done in various ways, some of the students worked alone and some created their Easter egg cups together with their parents. Some parents proposed scaffoldings to their children, where they first measured the size of the egg, chose different shapes and designed an egg cup on paper (Figure 3).

Students worked either on iPads or on computers (Figure 4). Some switched during the design process, as they were not used to design with a touch screen. This was mainly the case for younger students aged 5 to 7. Parents also reported that to support their children they needed aid with some manipulations in the digital modulation software (e.g., rotations

Fig. 3 Student measuring before starting to design

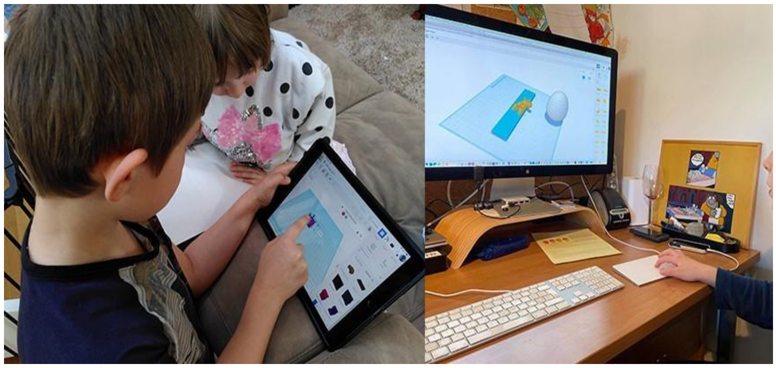

Fig. 4 Design realized on tablets and computers

and size adjustments of the shapes). During the challenge, we collaborated with some parents to assist their use of Autodesk® Tinkercad® and to explain how to place or move different objects in the design. Further, we supported them with resources on mathematical modelling (e.g., geometric shape families, measurements instruments, construction issues). Neumann (2018, p. 1661) describes in relation to parents and young students where “children may be developing experience and skills in operating tablets at home, age and technical ability are important factors to consider when supporting younger children’s interactions with tablets”. Parents tended to adapt their scaffolding to the age and estimated technical ability of their child (Kermani & Brenner, 2000).

Although not all students and parents used all features of AR in Autodesk® Tinkercad® or GeoGebra 3D Graphing Calculator, we got replies from some parents that it was beneficial to control the design through augmented reality (Figure 5). One could argue that the use or not use of AR was related to the parents’ scaffolding strategies (cognitive, affective, or technical) and therefore not be used in every setting. This seems to be similar to the teaching in class. Parents in a teaching role seem to make their own choices in their methodology to conduct the remote teaching. These choices could be related to their own skill set, to their perception of mathematics courses or their teaching experiences so far.

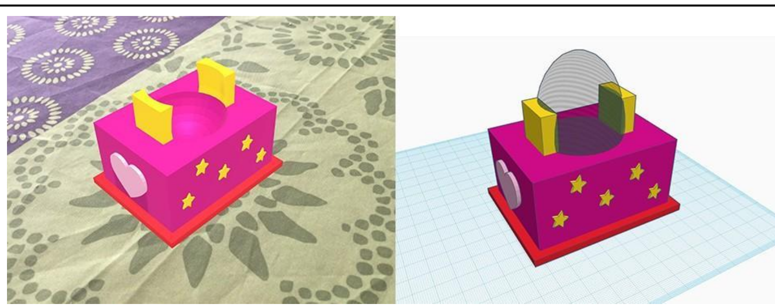

Fig. 5 The use of AR in the design

Fig. 6 3D Prints of the student's designs

At the end of the challenge our group printed out the different designs (Figure 6). However, unlike the in-school experience at this stage, students could not optimize their designs, rearrange them, or supervise the printing process. This element was only possible if students had their own 3D printer at home. Thus, unfortunately, in some of the designs, the egg cups were not equilibrated once they got printed and were not able to accomplish their function and hold the egg. In these cases, we collaborated via a shared link with parents and students to aid them to modify their designs. Still, most of the designs have been designed in a manner that they were possible to complete and print without any difficulties.

In the next sections we will outline our data analyses and results. As described earlier, in our analyses we mainly focus on parents' perspectives as during the confinement parents played a major role in the teaching of their children and we aimed to explore these educational dynamics in this paper.

Method

Grounded theory approach

To carry out an exploratory study on parents' perspectives in confinement, we decided to follow a grounded theory approach by Corbin and Strauss (1990, 2014). This qualitative approach was chosen as it allowed us to develop new theoretical aspects through uncovering concepts and relations in this new learning setting. In this exploratory study, we wanted to identify how parents were dealing with their new role as students' helpers in mathematical modelling and how working on our tasks challenged them. According to grounded theory approaches, we used the three different coding methods (open, axial, and selective) for our analyses.

Data collection

We collected data on the use of AR, digital and physical modelling in remote teaching through parent's perspectives in various ways. We conducted, four semi-structured interviews with parents, three semi-structured interviews with elementary school teachers, one semi-structured interview with a school leader, eight semi-structured interviews with students from age 7 to 13, and received data from open questions (e.g., How did you as parents assist your child? Which parts in the mathematical modelling process were difficult for you and your child? How was the motivation of your child during the modelling?) in an online questionnaire with the 13 parents at the end of the remote teaching. Adding to this, we analysed emails, conversations and messages on Microsoft Teams and WhatsApp groups that we got from parents.

Methods of data analysis

Thus, we coded and compared the collected data (i.e., answers in the semi-structured interviews, response from the online questionnaire, observations, Microsoft Teams conversations and others) in iterative cycles. This process of constant comparative data analysis and re-coding (Corbin & Strauss, 2014) allowed us to identify themes and a conceptual framework on parents' perspectives. Thus, rereading of transcripts, questionnaire responses, and coding highlighted a set of different themes that were dominating the data.

Results

We were able to identify six overall themes:

- *Theme 1:* Perceptions of mathematics courses and teaching
- *Theme 2:* Motivations of students
- *Theme 3:* Scaffolding and parent-assisted teaching in mathematical modelling
- *Theme 4:* Technical knowledge of parents
- *Theme 5:* Support and guidance
- *Theme 6:* Influences on learning and teaching behaviours

We used these themes as a conceptual framework and sorted codes from the studies into the different themes. In the upcoming paragraphs, we will outline these different themes that emerged from the data analyses with explanations and examples.

Theme 1: Perceptions of mathematics courses and teaching

Perception on mathematical modelling in Geometry emerged from almost every conducted interview. Thus, these perceptions are constantly compared to existing beliefs and beliefs from new experiences (Diego-Mantecón et al., 2019). The use of new technologies, such as AR or the digital and physical mathematical modelling was perceived more or less important in the learning of mathematics. Parents indicated that in the beginning AR, digital and physical mathematical modelling were valuable assets to foster creativity and visual experiences of students (e.g., show a geometric shape in 3D, try to place different shapes with AR on a given object); however, it should have been more connected to the textbooks and schedules of classes. Parents also indicated that written calculations (e.g., calculating the result of a volume of geometric shapes is seen as more important than understanding how to modulate these shapes) seemed to be a priority focus in mathematics and that mathematical modelling in Geometry was perceived as less important. Hence, according to some parents' perceptions, tasks shouldn't interrupt traditional teaching, but should be offered as optional work for some students. These parents reported that "our children already have to do much homework and calculations and we need to help them with these" or "the schedule in school is very tight and we need to follow the textbooks and exercises required by the teacher" or "children need to solve calculations without technology".

Other parents affirmed that they wanted technologies such as AR, digital and physical mathematical modelling more often in remote and class teachings. Further, these parents accorded higher importance to mathematical modelling and active transfer of mathematical skills, than executing simple operations on volumes or areas. "Our boy was able to use his mathematical knowledge to create the design. This was more challenging than the textbook exercises or the long calculation exercises in the homework", or "It is preparing the children for secondary school and the skills they will need for a future job", parents reported. Referring to the mathematical modelling framework (Blum & Leiß, 2007), these parents described how their children went from real-world information through the different steps of modelling (e.g., combine shapes, rearrange their volumes, compare different models).

Overall, perceptions of mathematics and teaching mathematical modelling in geometry seem to play an important role in how parents accept new technologies in their children's mathematics learning. Similar to findings of Tapia and Marsh (2004), changes could be attributed to "confidence" and "enjoyment". Hence, a positive learning experience of their children, could support parents in time to understand the value of these technologies. Hence, we should reflect on future tasks and how we could promote and explain the benefits of these technologies in mathematical modelling as well as to allow parents and students to exchange these experiences.

Theme 2: Motivation of students

Parents reported high motivations for their children during working with AR, digital and physical mathematical modelling, comparable to findings of Bacca Acosta et al. (2014). There had been no differences in reported motivation, whether children were low or high performing in mathematics courses. Unlike regular remote teaching, without the use of these technologies, parents did not need to insist and give continuous scaffolding to keep their children doing their learning tasks. Students seemed to be motivated by the hands-on activities and the active creation tasks (Craig, 2000). Further, students seemed to be more self-confident in their mathematical modelling and studied more autonomously. Positive self-esteem could even be related to increased academic performances (Alkhateeb, 2014). Further, parents reported that “the challenge made them really curious”, “they (children) were motivated because of the creative uses of maths” and “he was motivated to do something in 3D which will be created in real”. The use of technologies connecting mathematical knowledge with real-world information to be able to create and design an object, was overall seen as highly motivating by parents during remote teaching. One could assume that the more a goal of a task relates to real-world situations, objects or problems the more parents and students seemed to be motivated in the execution of the task (Liu et al., 2019; Singer et al., 2013). Accordingly, future remote teaching, whenever it is possible, should consider connecting learning to student’s real-world information within the students’ living environments.

Theme 3: Scaffolding and parent-assisted teaching

During remote teaching, parents adopted a similar role as a teacher and employed different strategies to motivate and guide their children during their learning processes. They assisted their children to solve tasks, by asking questions, giving hints, or motivating them with various strategies. Similarly, as teachers in classes, parents used different forms of scaffoldings, where a scaffolding means to provide assistance when children require assistance in solving tasks (Pressley et al., 1996). Comparable to findings of Hirschland (2008), parents tried to adapt scaffoldings to their children’s estimated skill levels or their requirements. We classified parents’ reported scaffoldings into three different categories, based on the framework outlined by Neumann (2018). Parents utilised cognitive, affective, and technical scaffoldings. “We did measurements with our child and asked questions about the shapes to be used” or, “we asked our daughter which shapes would fit well together and how they can be built up”, where these typical cognitive scaffoldings were reported by parents. These cognitive scaffoldings had been performed in a similar way to the Dienes’ theory of mathematics (Lieban & Lavicza, 2019) and referred essentially to the different steps in the mathematical modelling process described by Blum (2013). Parents supported students in simplifying real-world information (e.g., size of the egg and negative

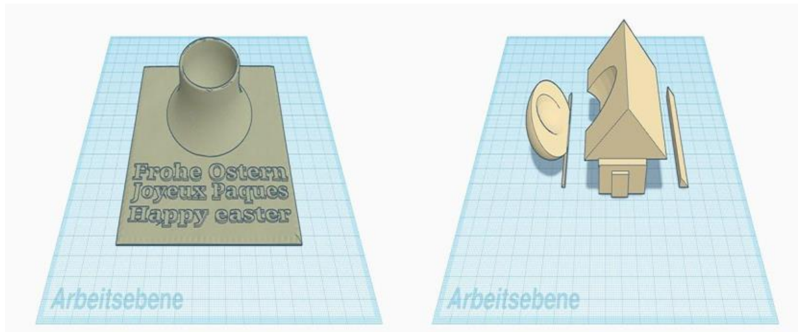

Fig. 7 From simple to complex Easter egg cups

print in the egg cup) and mathematization (e.g., recognize shapes and their properties, family of shapes or identify measurement concepts). Further, from the data, we know that parents engage students in iterative process design cycles, while supporting complexity of the modelling process (e.g., variations of sizes, new combinations of shapes and transformations of given objects), by creating a cognitive "path with questioning" to help their children to reach a high-quality design. Moreover, the learning paths were segmented into subtasks, from measuring shapes to designs with technologies, and students were guided through them with their parents' questions. These cognitive scaffoldings could be reproduced from parents' own scholastic experiences, similar to findings of Crook (1997). Thus, designs from the students reached from construction close to standard egg cups, up to complex designs, which were not distinguishable at first sight as an egg cup (Figure 7).

As for effective scaffoldings, parents encouraged their children and offered positive feedback to support their designs. During the interviews, parents reported that their children's motivation was high and that they could easily give positive feedback. Some parents reported that it came "very naturally" and it was "easy to support him, because he did so well". Affective scaffoldings in such remote teaching situations seemed to strengthen the relationships between parents and their children and influence teaching and learning positively (Price et al., 2011). Positive strengthening of these learning relationships, however, was reported rarely the case for the traditional remote teaching activities and assignments they did so far without these technologies. Parents admitted "this 3D task is more comfortable to do, than just to fill out pages of written calculations".

Technical scaffolding consisted mostly of connecting modelling software and design tasks. Parents needed to guide their children through software functions such as "rotating the design", "drag and drop shapes", "place different layers" and "uses of tactile modelling on iPad". These technical scaffoldings were more present in parents-student's settings with younger students aged 5 to 8. This was similar to the findings of Neumann and Neumann (2014), who agree that higher technical scaffolding could improve learning with educational technologies in this young age and to Wood et al. (2016) assuming that the intensity of scaffoldings is higher in a younger age. Based on our findings, parents' choice of scaffoldings and quality of scaffoldings should be supported in possible future remote teaching situations and teaching in general, after the end of the confinement. The high influence on students learning through parents' scaffoldings (Bodrova & Leong, 1998) should not be left as the responsibility of parents but should be supported through training or guidance to ensure that every child receives better learning scaffoldings.

Theme 4: Technical knowledge of parents

Educational technologies can be categorised, according to Means (1994), into four different categories: tutors, exploring, subject tools and communication enhancers. Each educational technology category requires different approaches and manipulations by the user. In our study, parents and students used Autodesk® Tinkercad® and GeoGebra 3D Graphing Calculator, which were the required technologies to explore, learned their interfaces, and decided how to use them with their children (Murray & Olcese, 2011). These technologies were rather new to both parents and students. Throughout the semi-structured interviews, parents indicated that in the regular course they used tutoring systems and communication applications; consequently, parents needed to rely on other skills than they used during traditional remote teaching. Parents with different technical knowledge expressed remarkably different concerns about their uses of technologies; thus, we were able to identify three different groups of parents in relation to their technical knowledge. In the first group of parents, we included parents who were highly familiar with design technologies, mostly because they had to use similar technologies in their professions or parents who were working within the educational system. These parents were able to identify manipulatives and were proficient with most aspects of required technologies. In the second group, we had those parents who indicated struggling with some manipulatives such as the rotation of the entire construction or adding of layers. These parents indicated that they did rely on their children's skills in technology to identify the needed manipulatives and/or asked for additional via email and obtained screenshots or tutorials. They were able to continue working on the task after receiving sufficient help. In the third group we placed parents who needed more support than the previous groups and having tutorials and screenshots were insufficient for them. These parents indicated that they had not worked with these kinds of technologies before and therefore encountered "general problems with technology". For these parents, we needed to set up additional assistance and we had to show manipulatives and processes for them in our school. Considering these technological differences among these three groups of parents we assumed that during remote teaching several parents would need extensive support. We set up additional support but realised that the latter two groups required even more assistance to be able to fully help their children. This is aligned with the study of Pacifici et al. (2006) on web-based training for parents. Learning from this experience we offered structured and personalised training for parents, afterwards in a similar study in kindergarten, and now we would be able to design better support. Nevertheless, further investigations and approaches would be needed in future studies.

Theme 5: Support and guidance

We offered several kinds of support and guidance for students and parents. Our support was based on multiple communication channels in order to ensure that parents with different technology expertise and Internet access could benefit from our support, similarly, as was described by Rothbaum et al. (2008). We communicated regularly with parents through telephone calls, emails, Microsoft Teams and WhatsApp. Parents could ask questions or ask for guidance to be able to assist their children. Initially, their main questions concerned technical issues like "how can we save a design", "where do I find the different shapes", "how can we add an object on another side of the shape" or "how can I take a picture of the design in AR". Later they mostly asked questions about how they should work with their children; they asked, "how much guidance should I provide" or "how much time should I let my children work with the technologies". According to the study of Neumann (2015) the time a child spends using educational technologies was from 20 to 80 minutes a day, while parents in our study estimated and asked for 30 to 60 minutes periods. We developed three video tutorials on technical manipulations, explained tasks on a webpage and in a podcast to be also used in our later studies. Furthermore, we offered additional support via email if parents came up with additional questions. Parents reported that "the shared link in the program helped to visualize the problems", "I think it is good to get some help through telephone calls or the Microsoft Teams account of children", or "When you showed it to me, I was able to show it to my child". Similar to the findings of Liao et al. (2017), parents increased their engagements with their children's learning processes through the offered support opportunities. Parents reported in interviews that not all of their issues would have required direct support from teachers, thus, we are working on developing communication channels among parents and encouraging community support. We will aim to analyse such community channels in our future studies. According to Shilling et al. (2013) peer support and community building among parents could have a positive effect on the entire educational process.

Theme 6: Influences on learning and teaching behaviours

Throughout our studies, we observed, and data analyses also supported, that parents and students were highly engaged and motivated while using AR, digital and physical modelling during their learning and teaching processes. We observed, as Selter and Zannetin (2019), that transfer of mathematical modelling and solving real-world situations, objects or problem tasks with digital and physical modelling influenced students' mathematical problem-solving strategies overall. Students changed the way they tried to solve tasks with technologies and tended to visualise and experiment more in their solutions. Their perception of mathematical tasks seemed to change and they utilised more creative strategies and different mathematical skills comparable to findings of Steen et al. (2006) and Moyer-Packenham et al. (2008). Parents were also experiencing new ways of supporting their children's mathematical learning and many were able to utilise skills from their professional lives. Some parents used knowledge from their jobs, "I am an architect and I work with similar programs; it was really fun to work with my son on this project", or some parents experimented with new types of learning "we were on a kind of expedition to create something together". Most parents tried to guide their children and motivate them, as we reported in the scaffolding theme. Their supports were more visual and thus more appropriate for the utilised technologies, "I asked my child to create a prototype and then we discussed it", or "we created all possible solutions in the program". The change of teaching compared to the experiences they had with more traditional tasks could be related to the aids of manipulatives, however, while using digital and physical modelling their experiences shifted toward more creative approaches similarly to findings of Sung et al. (2015). Parents also reported that "we argued about the possible solutions and tried different ways" and "we modulated the geometric shapes many times and discussed them to get the right design". Thus, parents and students used process skills in mathematical thinking, to solve the required tasks, which could be seen as a strong predictor of performance growth in mathematics courses (Saçkes, 2013). Students were able to connect their experiences with other subjects and took different perspectives for complex solution strategies, according to some of the parents. This was further enhanced by teachers' and parents' scaffoldings and heightened students' motivations, technical and content knowledge. It was clear that parents had extensive influences on the students' learning behaviours and on our experiments.

Conceptual framework and discussion

After regrouping the codes into different themes, we were able to identify interactions that were influencing remote teaching with AR, digital and physical mathematical modelling for parents. Figure 8 outlines our conceptual framework. Perceptions of mathematical modelling in Geometry made parents more or less open-minded for the use of these technologies in remote teaching, which we assume is strongly related to their beliefs from past experiences with mathematics (e.g., Diego-Mantecón et al., 2019). Parents who were more likely accepting technologies were the ones who were able to observe connections between the mathematical curriculum and the overall usefulness of technologies in life as

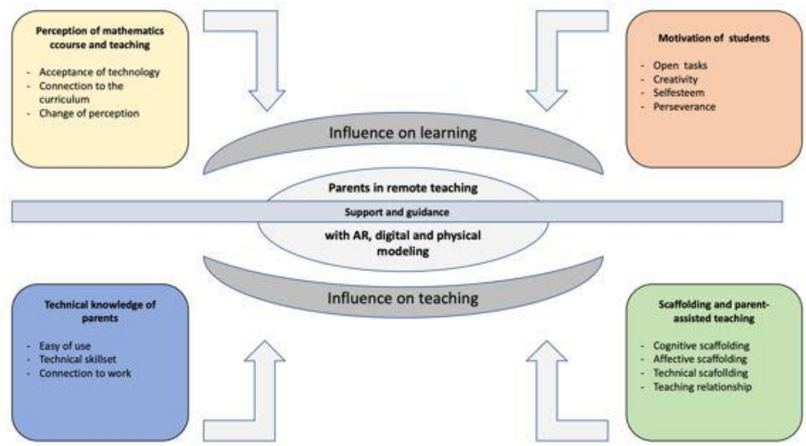

Fig. 8 Conceptual Framework on parents' perspectives in teaching with AR, digital and physical modelling

well as in learning. Of course, the results of the study are only of limited significance, as the number of participants was relatively small.

Those parents who were uncertain about the uses of technologies in classes did not believe that it could be useful for learning; however, their perceptions and openness changed after working with their children using AR, digital and physical modelling. Thus, based on these findings we are developing new kinds of tasks and support for students, teachers and parents, to work together to stimulate positive perceptions by strengthening motivations, enjoyment, and confidence in these mathematical modelling tasks (Tapia & Marsh, 2004). Furthermore, motivation of students appeared to be one of the key factors for strengthening relationships between parents and students in remote teaching. Similarly, as Craig (2000), we assumed that mathematical modelling with real-world situations, objects and problems could raise the motivations of students. AR, digital and physical mathematical modelling served as important elements of our task and pedagogy designs during the remote teaching, as it supported, based on our observations, transitions from one mathematical modelling step to the next. Comparable to the findings of Price et al. (2011), adequate tasks and teaching approaches not only strengthened relationships of students and their parents, but also positively influenced students' performances in their courses. Hence, students could gain mathematical modelling skills, by using technology, outside the classroom, with their parents. Moreover, students showed perseverance in solving tasks and gained higher self-esteem during their learning process according to reports of parents and consistent with other research (Hall, 2006). Findings, concentrating on students and parents will be reported in further papers. Nevertheless, in future studies, we will investigate long-term effects, while students return to school and continue using AR, and digital and physical mathematical modelling and explore students' knowledge on national tests.

In remote teaching, parents scaffolded tasks with cognitive, affective, and technical approaches as described in the work of Neumann (2018). Similarly, to students' learning in non-digital activities (Aram, 2008), we were able to identify some positive effects of these scaffolding approaches with AR, digital and physical modelling. Parents often utilised their own experiences with these technologies during their work with their children. While supporting parents, we had to attend a variety of requests and encourage parents to utilise their own experiences and learn new skills to be able to better help their children. We noticed that peer support was important and in future studies closer attention should be paid on developing communication channels and building a support community. Findings also suggested that shifting from traditional tasks and learning approaches was beneficial, increasing parents' motivations and dedications in their involvement in children's learning processes. Students were able to apply mathematical knowledge on real-world information, with the guidance of their parents. The designs had different complexities and parents guided and supported them in many ways. Overall, in our study during confinement, parents played a crucial role in remote teaching and in utilising new technologies while learning mathematics. In comparison to our classroom teaching phases, parents became assistant teachers and gatekeepers who can engage students in active learning processes and experimentations with new technologies. However, since parents were not professional teachers, they needed new kinds of assistance and support to empower their potentials, but it became clear that the roles of parents, especially during home schooling, are crucial for students' successes and must be considered in future studies not only during home learning situations, but also involving them in school and mixed learning activities.

Abbreviations

3D: Three dimensional; AR: Augmented Reality; COVID-19: Official name for the disease caused by the SARS-CoV-2 (2019-nCoV) coronavirus.

Endnotes

[a] We collected permission from all participants shown on the pictures.

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Authors' contributions

Ben Haas edited all sections of the first manuscript. Zsolt Lavicza reviewed and edited the methodology and methods sections. Yves Kreis consulted on the research, edited the revised version, and formatted the final version.

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Declarations

Competing interests
The authors declare that they have no competing interests.

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References

Alkhateeb, H. M. (2014). Self-esteem and mathematics achievement of elementary Qatari students. *Psychological Reports*, *114*(3), 971–973E. https://doi.org/10.2466/11.07.PR0.114k28w1

Aram, D. (2008). Parent–child interaction and early literacy development. *Early Education and Development*, *19*(1), 1–6. https://doi.org/10.1080/10409280701838421

Bacca Acosta, J. L., Baldiris Navarro, S. M., Fabregat Gesa, R., & Kinshuk, K. (2019). Framework for designing motivational augmented reality applications in vocational education and training. *Australasian Journal of Educational Technology*, *35*(3), 102–117. https://doi.org/10.14742/ajet.4182

Bacca Acosta, J. L., Baldiris Navarro, S. M., Fabregat Gesa, R., Graf, S., & Kinshuk, K. (2014). Augmented reality trends in education: A systematic review of research and applications. *Journal Educational Technology & Society*, *17*(4), 133–149.

Berdik, C. (2017). Kids code their own 3d creations with new blocks-based design program. *Tech Directions*, *76*(9), 23–24.

Billinghurst, M., & Duenser, A. (2012). Augmented reality in the classroom. *Computer*, *45*(7), 56–63. https://doi.org/10.1109/MC.2012.111

Blum, W. (2013). Mathematical modeling: How can students learn to model? *Journal of Mathematics Education at Teachers College*, *Proceedings: Conference on Mathematical Modeling*, 54–61. https://doi.org/10.7916/jmetc.v0i0.662

Blum, W., & Leiß, D. (2007). How do students and teachers deal with modelling problems? In C. Haines, P. Galbraith, W. Blum & S. Khan (Eds.), *Mathematical Modelling (ICTMA 12): Education, Engineering and Economics* (pp. 222–231). Woodhead Publishing.

Bodrova, E., & Leong, D. J. (1998). Scaffolding emergent writing in the zone of proximal development. *Literacy Teaching and Learning, 3*(2), 1–18.

Cai, S., Liu, E., Yang, Y., & Liang, J. (2019). Tablet-based AR technology: Impacts on students' conceptions and approaches to learning mathematics according to their self-efficacy. *British Journal of Educational Technology*, *50*(1), 248–263. https://doi.org/10.1111/bjet.12718

Clements, D. H., & Sarama, J. (2011). Early childhood teacher education: The case of geometry. *Journal of Mathematics Teacher Education*, *14*(2), 133–148. https://doi.org/10.1007/s10857-011-9173-0

Corbin, J. M., & Strauss, A. L. (2014). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (4th ed.). SAGE Publications, Inc.

Corbin, J., & Strauss, A. (1990). Grounded theory research: Procedures, canons and evaluative criteria. *Zeitschrift für Soziologie: ZfS*, *19*, 418–427.

Craig, D. V. (2000). Technology, math, and the early learner: Models for learning. *Early Childhood Education Journal*, *27*(3), 179–184. https://doi.org/10.1007/BF02694232

Crook, C. J. (1997). *Cultural practices and socioeconomic attainment: The Australian experience*. Greenwood Press.

Diego-Mantecón, J. M., Blanco, T. F., Chamoso, J. M., & Cáceres, M. J. (2019). An attempt to identify the issues underlying the lack of consistent conceptualisations in the field of student mathematics-related beliefs. *PLoS ONE*, *14*(11), 1–19. https://doi.org/10.1371/journal.pone.0224696

Dienes, Z. P. (1960). *Building up mathematics*. Hutchinson Educational. https://archive.org/details/buildingupmathem00dien

González, N. A. A. (2015). How to include augmented reality in descriptive geometry teaching. *Procedia Computer Science*, *75*, 250–256. https://doi.org/10.1016/j.procs.2015.12.245

Haas, B., Kreis, Y., & Lavicza, Z. (2020). Connecting the real world to mathematical models in elementary schools in Luxemburg. In R. Marks (Hrsg.), *Proceedings of the British Society for Research into Learning Mathematics: Bd. 40 (2)* (S. 1–6). British Society for Research into Learning Mathematics. https://bsrlm.org.uk/wp-content/uploads/2020/10/BSRLM-CP-40-2-06.pdf

Hall, A. L. (2006). *The relationship between academic achievement, academic performance and self-esteem of high school juniors at a public high school in central Florida*. Capella University.

Hirschland, D. (2008). *Collaborative intervention in early childhood: Consulting with parents and teachers of 3-to 7-year-olds*. Oxford University Press USA.

Hohenwarter, M., Borcherds, M., Ancsin, G., Bencze, B., Blossier, M., Elias, J., Frank, K., Gal, L., Hofstaetter, A., Jordan, F., Karacsony, B., Konecny, Z., Kovacs, Z., Kuellinger, W., Lettner, E., Lizelfelner, S., Parisse, B., Solyom-Gecse, C., & Tomaschko, M. (2020). *GeoGebra 3D Graphing Calculator* (6.0.636.0). International GeoGebra Institute.

Kermani, H., & Brenner, M. E. (2000). Maternal scaffolding in the child's zone of proximal development across tasks: Cross-cultural perspectives. *Journal of Research in Childhood Education*, *15*(1), 30–52. https://doi.org/10.1080/02568540009594774

Lavicza, Z., Haas, B., & Kreis, Y. (2020). Discovering everyday mathematical situations outside the classroom with MathCityMap and GeoGebra 3D. In M. Ludwig, S. Jablonski, A. Caldeira & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age: Proceedings of the ROSETA Online Conference in June 2020* (Vol. 6, pp. 23–30). WTM.

Liao, C. C. Y., Cheng, H. N. H., Chang, W.-C., & Chan, T.-W. (2017). Supporting parental engagement in a BYOD (Bring Your Own Device) school. *Journal of Computers in Education*, *2*(4), 107–125.

Lieban, D. (2019). *Exploring opportunities for connecting physical and digital resources for mathematics teaching and learning* [Dissertation (PhD), Johannes Kepler Universität Linz]. http://epub.jku.at/obvulihs/4555198

Lieban, D., & Lavicza, Z. (2019). Instrumental genesis and heuristic strategies as frameworks to geometric modeling in connecting physical and digital environments. *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education (CERME11)*, 1–8.

Liljedahl, P., Santos-Trigo, M., Malaspina, U., & Bruder, R. (2016). Problem solving in mathematics education. In P. Liljedahl, M. Santos-Trigo, U. Malaspina & R. Bruder (Eds.), *Problem solving in mathematics education* (pp. 1–39). Springer.

Liu, E., Li, Y., Cai, S., & Li, X. (2019). The effect of augmented reality in solid geometry class on students' learning performance and attitudes. In M. E. Auer & R. Langmann (Eds.), *Smart industry & smart education* (Vol. 47, pp. 549–558). Springer.

Means, B. (1994). *Technology and education reform: The reality behind the promise*. Jossey-Bass.

MENFP. (2011). *Plan d'études: École fondamentale*. Ministère de l'Éducation nationale et de la Formation professionnelle. http://www.men.public.lu/catalogue-publications/themes-transversaux/cen/cens/plan-etudes/fr.pdf

Moyer-Packenham, P. S., Salkind, G., & Bolyard, J. J. (2008). Virtual manipulatives used by K-8 teachers for mathematics instruction: The influence of mathematical, cognitive, and pedagogical fidelity. *Contemporary Issues in Technology and Teacher Education*, *8*(3), 202–218.

Murray, O. T., & Olcese, N. R. (2011). Teaching and learning with iPads, ready or not? *TechTrends*, *55*(6), 42–48. https://doi.org/10.1007/s11528-011-0540-6

Nakamura, J., & Csikszentmihalyi, M. (2014). The concept of flow. In M. Csikszentmihalyi (Ed.), *Flow and the foundations of positive psychology: The collected works of Mihaly Csikszentmihalyi* (pp. 239–263). Springer Netherlands. https://doi.org/10.1007/978-94-017-9088-8

NCTM. (1999). *NCTM principles and standards for school mathematics*. National Council of Teachers of Mathematics (NCTM).

Neumann, M. M. (2015). Young children and screen time: Creating a mindful approach to digital technology. *Australian Educational Computing*, *30*(2). http://journal.acce.edu.au/index.php/AEC/article/view/67

Neumann, M. M. (2018). Parent scaffolding of young children's use of touch screen tablets. *Early Child Development and Care*, *188*(12), 1654–1664. https://doi.org/10.1080/03004430.2016.1278215

Neumann, M. M., & Neumann, D. L. (2014). Touch screen tablets and emergent literacy. *Early Childhood Education Journal*, *42*(4), 231–239. https://doi.org/10.1007/s10643-013-0608-3

Ng, O., & Chan, T. (2019). Learning as making: Using 3D computer-aided design to enhance the learning of shape and space in STEM-integrated ways. *British Journal of Educational Technology*, *50*(1), 294–308. https://doi.org/10.1111/bjet.12643

Pacifici, C., Delaney, R., White, L., Nelson, C., & Cummings, K. (2006). Web-based training for foster, adoptive, and kinship parents. *Children and Youth Services Review*, *28*(11), 1329–1343. https://doi.org/10.1016/j.childyouth.2006.02.003

Pressley, M., Hogan, K., Wharton McDonald, R., Mistretta, J., & Ettenberger, S. (1996). The challenges of instructional scaffolding: The challenges of instruction that supports student thinking. *Learning Disabilities Research and Practice*, *11*(3), 138–146.

Price, M., Handley, K., & Millar, J. (2011). Feedback: Focusing attention on engagement. *Studies in Higher Education*, *36*(8), 879–896. https://doi.org/10.1080/03075079.2010.483513

Rothbaum, F., Martland, N., & Jannsen, J. B. (2008). Parents' reliance on the web to find information about children and families: Socio-economic differences in use, skills and satisfaction. *Journal of Applied Developmental Psychology*, *29*(2), 118–128. https://doi.org/10.1016/j.appdev.2007.12.002

Saçkes, M. (2013). Children's competencies in process skills in kindergarten and their impact on academic achievement in third grade. *Early Education and Development*, *24*(5), 704–720. https://doi.org/10.1080/10409289.2012.715571

Selter, C., & Zannetin, E. (2018). *Mathematik unterrichten in der Grundschule: Inhalte - Leitideen - Beispiele*. Friedrich Verlag.

Shilling, V., Morris, C., Thompson-Coon, J., Ukoumunne, O., Rogers, M., & Logan, S. (2013). Peer support for parents of children with chronic disabling conditions: A systematic review of quantitative and qualitative studies. *Developmental Medicine and Child Neurology*, *55*(7), 602–609. https://doi.org/10.1111/dmcn.12091

Sinclair, N., & Bruce, C. D. (2015). New opportunities in geometry education at the primary school. *ZDM*, *47*(3), 319–329. https://doi.org/10.1007/s11858-015-0693-4

Singer, F. M., Ellerton, N., & Cai, J. (2013). Problem-posing research in mathematics education: New questions and directions. *Educational Studies in Mathematics*, *83*(1), 1–7. https://doi.org/10.1007/s10649-013-9478-2

Steen, K., Brooks, D. W., & Lyon, T. (2006). The impact of virtual manipulatives on first grade geometry instruction and learning. *Journal of Computers in Mathematics and Science Teaching, 25*(4), 373–391.

Sung, Y.-T., Shih, P.-C., & Chang, K.-E. (2015). The effects of 3D-representation instruction on composite-solid surface-area learning for elementary school students. *Instructional Science*, *43*(1), 115–145.

Szucs, D., Devine, A., Soltesz, F., Nobes, A., & Gabriel, F. (2013). Developmental dyscalculia is related to visuo-spatial memory and inhibition impairment. *Cortex*, *49*(10), 2674–2688. https://doi.org/10.1016/j.cortex.2013.06.007

Tapia, M., & Marsh, G. E. (2004). An instrument to measure mathematics attitudes. *Academic Exchange Quarterly*, *8*(2), 16–22.

Wood, E., Petkovski, M., De Pasquale, D., Gottardo, A., Evans, M. A., & Savage, R. S. (2016). Parent scaffolding of young children when engaged with mobile technology. *Frontiers in Psychology*, *7*. https://doi.org/10.3389/fpsyg.2016.00690

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