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COLLABORATIVE MATHEMATICAL INQUIRY WITH AUGMENTED REALITY

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In this paper we describe and reflect on the design of a mathematical learning activity developed in collaboration between teachers, researchers and technical developers. By making use of augmented reality (AR) as a technology supporting augmentation of a real-world projection with computer-gene-rated images, we have designed an activity that promotes unique action and learning trajectories. These trajectories require the learners to engage in interactive-constructive actions that involve and stimulate the development of their self-regulatory skills by inviting them to vary and coordinate across the contextual affordances of the technologies and the physical resources in the classroom. Our learning activity is designed as a collaborative guided inquiry, implemented in a regular classroom and involved mathematical problem solving in relation to the geometric concept of scale. In order to successfully complete the activity, the learners are challenged to coordinate affordances from three distinct referential contexts by involving physical and virtual artifacts. In the design process, we identify critical aspects of the activity and embed affordances for corresponding scaffolding actions which turn out to play a crucial role when the activity is implemented with a group of four 15-year-old students. Although the AR technology has served us well in developing this particular activity, this specific technology appears to have limited applicability in mathematics education beyond geometry. We recommend that future research efforts move beyond AR and consider the broader context of embodied design with tangible user interfaces, that have recently shown great potential for the design of innovative activities for the learning of mathematics.

Keywords: Augmented reality; design-based research; mathematics education; technology-enhanced learning; inquiry-based learning; self-regulation; tangible user interfaces.

1. Introduction

1.1. *General background*

The author has been involved in several collaborative design-based research projects that put focus on developing, implementing and evaluating curriculum-based mathematical learning activities supported by technologies (Sollervall et al., 2011; Sollervall, Otero, Milrad, Johansson, & Vogel, 2012) including projects with augmented reality (Nilsson, Sollervall, & Spikol, 2010; Sollervall, Nilsson, & Spikol, 2010). The current paper reports on such a team effort involving researchers in mathematics education, researchers in media technology, technical developers and mathematics teachers, who set out to investigate how augmented reality – as a real-world projection augmented with computer-generated images – can be used to stimulate students' collaborative learning processes regarding the mathematical concept of scale. Inspired by recent trends in the Swedish national school curriculum, the team decided to attend specifically to stimulating students' strategic thinking and decision-making when interacting with peers and with the activity.

Designing activities intended to support students in achieving specific learning objectives while interacting with peers, technology, other artifacts, and possibly a teacher, calls for careful design considerations regarding the selection of technologies and artifacts as well as their pedagogical integration in the activity. Furthermore, introducing activities that put high demands on the students' self-regulatory skills calls for preparing critical scaffolding actions to be provided on a need basis during implementation of the activity. Due to the complexity of these design considerations, we have recently pursued research directed at consolidating the theoretical and methodological foundations we have identified as useful underpinnings for curriculum-based collaborative design-based research (Sollervall & Milrad, 2012). In this paper, we will make use of these theories and methodologies as we analyze the design process and the learning outcomes of a collaboratively designed learning activity involving augmented reality.

1.2. *Rationale and objectives*

In the current paper, we account for a collaborative design effort where we develop, implement and test a curriculum-based mathematical learning activity supported by augmented reality (AR) and involving the mathematical concept of scale. We will illustrate how AR affords the provision of unique opportunities for stimulating students' collaborative inquiries in the designed activity and show how these opportunities are unfolded during an implementation of the activity with a group of four 15–year-old students. Furthermore, we illustrate how addressing critical aspects of the activity in the design process, by embedding corresponding affordances for scaffolding, facilitates the teacher's orchestration of the activity and the students' successful completion of their inquiries.

2. Theoretical and Methodological Considerations

In this section, we account for theoretical and methodological considerations regarding collaborative design-based research, inspired by co-design and scenario-based design with focus on hypothetical learning trajectories involving artifacts and their affordances. Furthermore, we discuss how technological, educational, and pedagogical affordances can be designed to provide opportunities for inquiry-based learning and self-regulation by scaffolding learners' goal-oriented actions during an implemented learning activity. The section ends with a methodological discussion about data collection and data analysis.

2.1. *On collaborative design-based research*

Design-based research (DBR) may be considered as a reaction against the dominating educational research tradition of outcome evaluation (Glass, 1976; Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003). By explicitly addressing the issue of educational improvement, DBR is committed to the iterative development of innovative learning activities (Drijvers, 2003). Each iteration of an activity allows for adjusting and improving its features in order to better support the intended learning objectives (Cobb et al., 2003). The preliminary design phase of each iteration involves negotiating a suggested preliminary activity with a prospective analysis, focusing on hypothetical learning trajectories (HLT) that support achieving the learning objectives (Sollervall & Milrad, 2012). An HLT may be described as a possible sequence of observable actions and a corresponding sequence of cognitive constructs, where the plausibility of a specific HLT depends on the knowledge base of the targeted students and the comprehensive learning environment for the activity (Sztajn, Confrey, Wilson, & Edgington, 2012).

The complex task of designing learning activities with technologies calls for organizing design teams that coordinate didactical subject expertise with technological expertise. Our design efforts are inspired by the co-design methodology (Roschelle & Penuel, 2006; Penuel, Roschelle, & Shechtman, 2007) and involve researchers in mathematics education, researchers in media technology, technical developers, as well as mathematics teachers. The teachers in our projects are recruited from local schools (in Sweden) with the particular role to assess and improve the relevance of proposed activities with respect to their own teaching practices and the Swedish national curriculum for compulsory school. In latter iterations of an activity, they become responsible for adapting the activity so it may be adopted and implemented by other teachers.

An important feature of collaborative team work is its potential for innovation. By joining forces and drawing on each other’s competencies, the team work can “empower and intellectually liberate its participants” (Gershon, 2009). Such a process of intellectual liberation is often characterized by improvisation, imagination, and messiness. While such features are generally not highly regarded in research, they may afford creative opportunities and stimulate explorations with the potential for innovation (ibid.).

The activity presented in this paper emerged from an invitation to a mathematics educator (the author of this paper) to participate in testing a learning activity involving mobile technologies in an outdoor setting, followed by an activity with AR in a regular classroom. Stimulated by this experience, the author could imagine how to exploit the AR technology to support unique mathematical learning trajectories. By exposing oneself to situations created by others, you may draw on your personal perspective and expertise to identify and analyze embedded features that may be further developed in order to further exploit their educational potential. This is exactly what happened in our case: the decision to pursue educational design with AR emerged more or less by chance, as a random result of unstructured exploration of an existing activity. Collaborative design-based research specifically addresses the creative challenge posed 20 years ago regarding the use of technologies: “major limitations of computer use in the coming decades are likely to be less a result of technological limitations than a result of limited human imagination and the constraints of old habits and social structures” (Kaput, 1992, p. 515).

Collaboration between stakeholders with a common interest but varied backgrounds and complementing competencies may reduce the influence of social and cultural constraints and stimulate creative explorations and innovative processes.

The involvement of team members with different research backgrounds also enables the productive networking of theories, specifically in the coordination of research traditions and the combination of research findings from different fields (Prediger, Bikner-Ahsbahs, & Arzarello, 2008). The different stakeholders contribute in different ways to the knowledge that guides and scaffolds the process of collaborative design. It cannot be expected that each individual team member should be acquainted with all aspects of the knowledge base. The discussions and negotiations of HLT between the team members are in our case guided by scenario-based design (SBD; Rosson & Carroll, 2002) as a methodology based on narratives that enable "rapid communication about usage possibilities and concerns among many different stakeholders" (Penuel et al., 2007). A key issue in our collaborative DBR efforts is the appropriation of artifacts to an activity, specifically deciding which artifacts should be *made* available in the learning environment and deciding how this environment should stimulate the students to *perceive* the opportunities for action that are mediated by these artifacts (Sollervall & Milrad, 2012). The narratives of SBD could be descriptive, such as "the actor puts a ruler next to a side of the square", but should preferably include an interpretation of the intention behind the action, such as "the actor measures the side of the square by using a ruler". In this example, the ruler is an artifact that affords measuring. In our approach to collaborative DBR, we have specifically attended to the negotiation of such affordances, which may be defined as opportunities for action, or "qualities that the environment offers an actor" (Kirschner, Strijbos, Kreijns, & Beers, 2004).

2.2. *On technology-enhanced and inquiry-based learning*

Affordances have been suggested as a natural design instrument for the design of technology-enhanced learning (TEL) activities, where the use of affordances answers the need for creating design solutions that involve a suitable matching between *technological* affordances that can be deployed on specific technological resources and affordance requirements of an activity with respect to educational goals (Bower, 2008). These latter *educational* affordances can either be pre-defined or, as in our case, emerging in a collaborative design process where a wide selection of technological affordances are discussed and negotiated between the stakeholders in the design team (Sollervall & Milrad, 2012). When the desired affordances have been identified, their deployment in the learning environment is considered in terms of *pedagogical* affordances and the appropriation of affordance-mediating artifacts. The pedagogical affordances include providing instructions and hands-on trials to support students' processes of instrumental genesis, where they learn to unfold the educational affordances by interacting with the available artifacts and thus transform them into instruments for learning (Verillon & Rabardel, 1995).

Affordances and mediating artifacts are general notions that apply to all forms of human activities including organized learning activities. Carefully selecting artifacts for a learning activity becomes a crucial aspect of supporting learners' successful engagement in explorative activities where they are expected to engage in self-directed inquiries and construct their own strategies and solutions. Herron (1971) makes a distinction between guided inquiry, where only the task is given and guidance is provided on a need basis, and structured inquiry where also the procedure is prescribed. A guided inquiry challenges the learner to be constructive and generative, while a structured activity only requires the learner to be active and follow instructions. Current research in psychology (Chi, 2011) strongly suggests a relation between overt behavior and learning gains, where interactive-constructive behavior (exploring, investigating and generating in collaboration with peers) is considerably more favorable than just being active in the classroom for example when solving problems in the textbook. These findings imply that teachers should offer activities where learners can be interactive and constructive, which naturally involve problematizing, being challenged, making discoveries, refining principles and applying them to answer questions and solving problems, all in the spirit of inquiry-based learning (Edelson, Gordin, & Pea, 1999). Such constructive activities require the learner to engage in self-regulatory processes related to the following phases of self-regulation: fore-thought, planning and activation; monitoring; control; reaction and reflection (Schunk, 2005). We interpret the results by Chi (2011) and Boaler (2012) as confirming the belief held by Pintrich (quoted in Schunk, 2005) that activities requiring a high degree of self-regulation have a positive influence on learners' achievements. While forethought, planning, and activation refer to cognitive skills, monitoring and control concern motivation and behavior, respectively, reaction and reflection concern the context (Schunk, 2005). Any mathematical activity involves processes and actions that are specific for the subject domain of mathematics. Learners are challenged to engage in mathematical processes of defining, representing, generalizing, and justifying, and to take action on products of these processes (Zbiek, Heid, & Blume, 2012). While defining (forethought, planning, and activation) and justifying (reaction and reflection) may be interpreted as instances of self-regulatory processes, representing addresses cognitive skills as well as behavior. Efficiently representing mathematical objects, treating and making conversions between representations are essential aspects of sense-making and problem solving in mathematics (Duval, 2006). Generalizing beyond a current context may be regarded as a form of reflection that requires identifying general structures that exist beyond the context and simultaneously discriminating context-specific features. While solving textbook tasks is often limited to mathematical actions, a mathematical inquiry stimulates the learners to engage in (generative, constructive) mathematical processes.

Inquiry-based learning (as well as problem-based, learner-centered, discovery, experiential, and constructive learning) is often implemented with minimal guidance from the teacher and has in such cases been shown to be less effective for novice learners than guidance specifically designed to support their cognitive processing (Kirschner, Sweller, & Clark, 2006). These findings imply that we should be restrictive about implementing open inquiries where the learners work without being monitored by a scaffolding agent. Regarding computer-mediated learning environments, research efforts have been directed at deploying flexible and learner-sensitive metacognitive scaffolding on technologies (Miao, Weinbrenner, Engler, Giemza, & Hoppe, 2011). In our case the learning activity is designed for a classroom environment where the blending of technologies with traditional resources calls for scaffolding strategies that involve not only embedded features of the activity but also carefully planned strategies for teacher guidance and peer scaffolding (Edelson et al., 1999; Wong, Looi, Boticki, & Sun, 2011). Embedding crucial scaffolds in the activity and its technologies makes the remaining scaffolding tasks less demanding for the teacher, so that the activity could possibly be implemented in a regular classroom with an ordinary teacher without requiring interventions by the research team (Wong et al., 2011). However, the teacher has to be informed about the possible scaffolding actions and how they relate to embedded features in the activity so that the learners can be effectively stimulated to unfold their affordances. We conclude that designing a guided inquiry should include a scaffolding strategy that addresses the design of embedded scaffolds as well as critical scaffolding actions, that the teacher may enact on a need basis to guide the learners' inquiries.

2.3. *On the collaborative design of technology-enhanced guided inquiries*

We now proceed to summarize our comprehensive strategy for the collaborative design of technology-enhanced activities for the learning of mathematics. The methodology of DBR frames an iterative design process where a preliminary activity is tested, analyzed, and adjusted in each iteration. The adjustment and improvement of preliminary learning activity involves a balance of creative ideas and expert knowledge. This balance may be achieved through collaboration between experts from relevant subject domains, in-service teachers and – when technologies are involved – also technical developers.

The communication and creative realization of design proposals are effectively supported by making use of narratives of students' hypothetical interaction with the learning environment in a fashion that can be followed and understood by all members of the research team. A crucial aspect of these narratives is to account for students' actions in terms of affordances that are (made) available in the learning environment through mediating artifacts. By prioritizing affordances instead of their mediating artifacts we open up for achieving creative design solutions, as we may consider the deployment of a desired affordance on several different artifacts in order to select an artifact that efficiently mediates that affordance in the comprehensive learning activity. Comparing the mediating efficiencies of different artifacts is a considerably more challenging task than performing an affordance analysis of a single artifact. We address this challenge by involving experts in the design team who can readily identify and propose artifacts that mediate specific affordances and engage in a collaborative comparison of the mediating efficiencies of these proposed artifacts. The assessment of an artifact's mediating efficiency is mainly based on judgments of accessibility, that is, to what extent the learners will perceive and be able to unfold the desired affordances, but also based on judgments of the teacher's capability to guide the learners to unfold these affordances.

In our design process, we appropriate specific artifacts that facilitate the students' unfolding of these affordances during the implemented activity. We also design embedded scaffolds and identify critical scaffolding actions that allow the teacher to stimulate the learners to unfold the affordances. Design issues are considered from a holistic perspective, since all design elements are interdependent and have to be considered not only as individual elements but also in relation to the comprehensive activity.

Our aim with the approach described above is to develop activities which stimulate the learners to engage in interactive-constructive processes which put demands on their self-regulatory skills and offer them opportunities to further develop these skills.

2.4. *Data collection and data analysis*

The empirical data presented in this paper concerns both the design of the activity (Section 3) and the outcomes of the implemented activity (Section 4). The entire design process, from the original idea to implementation, encompassed a time period of roughly six months and involved three researchers, one technical developer, and two mathematics teachers. In Section 3, the design process is described in terms of narratives and analyzed with respect to considerations and provision of affordances and scaffolding features for the students' continued engagement in the goal-oriented mathematical inquiry.

The data from the implemented activity has previously been analyzed by using Intentional Analysis (Nilsson et al., 2010 ; Sollervall et al., 2010). Intentional Analysis allows to investigate how learners contextualize a task by making use of and combining contextual elements of conceptual, situational, and cultural nature (ibid.). Such an analysis aims to describe and explain students' actions rather than predicting the quality of learning resulting from these actions. In this paper, we have instead interpreted the same empirical data with respect to inquiry and self-regulation, thus implicitly addressing cognitive processes that have proven to be favorable for learning (Schunk, 2005; Chi, 2011). With this approach, we address (offered and unfolded) learning opportunities in the activity by identifying and describing how learners engage in (hypothesized as well as realized) interactive-constructive actions. Furthermore, this approach allows us to efficiently align the a posteriori outcome analysis (Section 4) with the apriori analysis of affordances and scaffolding features (Section 3).

The data was collected during a 30 minute long session where four Swedish students, all 15 years old, collaborated in solving the activity guided by one of the researchers. The students were used to working together in other school subjects than mathematics. They volunteered when the teacher asked the class who would like to participate in solving a mathematics problem with new technologies. The session was videotaped and additionally audio recorded. The videotapes and audio recordings were fully transcribed.

3. Design of the Activity

The current activity emerged as a spin-off from an on-going project in collaboration between researchers in media technology and high school mathematics teachers. The author was invited, as a researcher in mathematics education, to observe and test integrated outdoor and indoor activities making use of mobile technologies, where the AR technology readily showed potential as an instrument for learning mathematics. In parallel to the on-going project, we initiated a second collaborative design project focusing on making use of the AR technology in a regular classroom. The activity developed in this second project has previously been reported at two research conferences in mathematics education, which respectively have put focus on the didactical grounding of the activity (Sollervall et al., 2010) and evaluation of outcomes from implementation with a group of four students (Nilsson et al., 2010). In this section, we proceed to give a comprehensive account for the design process with focus on the provision of affordances for the students' engagement in goal-oriented mathematical inquiries.

3.1. *The AR technology and embodied design*

An augmented reality (AR) system is a technology that runs in real time and allows for mixing real-world images with computer-generated images (Milgram & Kishino, 1994). Although Sutherland already in the 1960's (1965) developed the first AR interface, it is only recently that researchers have explored its potential uses for formal education (Zhou, Duh, & Billinghurst, 2008). The 'Studierstube' is an example of an AR application for mathematics education, making use of head mounted displays allowing users to collaboratively view 3D geometric models from different perspectives in order to develop the users' spatial abilities and understanding of spatial features of geometric figures (Kaufmann & Schmalstieg, 2003). The Studierstube makes use of an optical see-through visual display that overlays images on a real-world perception, as opposed to video see-through that overlays images on a digitized image of the real world (van Krevelen & Poelman, 2010). The video see-through approach puts lesser demands on technological equipment and may be readily deployed on either an ordinary laptop with webcam or a hand-held device. What needs to be added is software supporting the AR technology, and prepared digitized images that may be used on to augment the real-world image. We chose to make use of the freeware BuildAR from www.hitlabnz.org, as one of several tools for creating AR applications (Yuen, Yaoyuneyong, & Johnson, 2011).

Searches on Google Scholar reveal a large number of educational research efforts relating to other subjects than mathematics such as chemistry, physics, and professional training. However, the only curriculum-oriented efforts with AR in mathematics education that appear in the research literature are further developments of the Studierstube and our own efforts (Nilsson et al., 2010; Sollervall et al., 2010), although recent and yet unpublished developments by Pierre Dillenbourg and colleagues with augmented paper for geometry learning appear to have potential for implementing AR in education (http://craft.epfl.ch/lang/en/PaperTangibleInterface). Additional searches in leading journals in mathematics education (Educational Studies in Mathematics, Journal of Mathematical Behavior) show no articles related to augmented reality.

Although there are few examples of AR applications in mathematics education, we may interpret the video see-through version of AR in a broader context of technological approaches with tangible user interfaces – paper tags or other physical objects – that replace expensive physical and electronic equipment. Two such examples are Lori Scarlatos' work on tangible user interfaces (2006) and Dor Abrahamson's embodied design approach (2011) that has resulted in the design of learning activities where learners enact proportionalities and explore mathematical concepts in probability.

3.2. *The technical and physical set-up for our activity*

The activity discussed in this paper involves collaboration between students and affords integration of real objects (and images of real objects) in the AR images. For these reasons, we chose a video see-through setup involving a laptop with webcam, connected to a projector facilitating the AR images to be shown on a whiteboard. The physical set-up for our activity (Figure 1, left pane) may be described as follows (Sollervall et al., 2010). A photograph of the participating students' home city from a bird's-eye view is laid down horizontally on a table. The webcam is placed just slightly above the table so that the map is projected on the whiteboard from a close to horizontal perspective. A movable small paper tag with a printed Quick Response (QR) code is placed on the table. The AR software supports showing any prepared image on top of the tag. In our case, we made use of self-prepared images of several buildings: the Turning Torso (a tall building in Malmö, Sweden), the Eiffel Tower, the London Eye, the local Cathedral (in Växjö, Sweden) and the Pyramid of Cheops, each connected with a unique QR code.

Using well-known buildings and a photograph with familiar landmarks was assumed to be beneficial for the participants' ability to evaluate whether their reasoning or a proposed solution seemed plausible (Nilsson et al., 2010).

When the webcam reads the QR code, the corresponding image of a building is shown on the whiteboard *but not on the table*. The tag may be moved to a new position on the table (Figure 1, right pane). If the webcam can identify the QR code at the new position, the image is shown on the whiteboard at the projected position of the tag. The

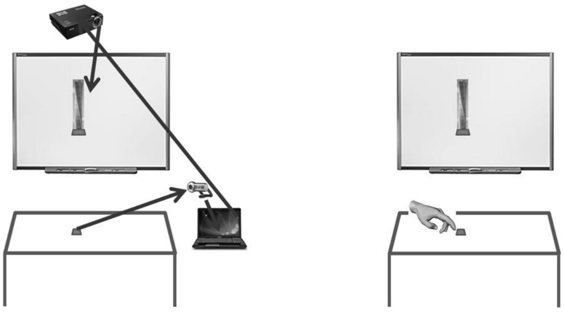

Figure 1. The technical set-up (left pane) and the physical set-up (right pane).

scale of the projected image is adjusted by the software, based on the size of the quadratic frame of the QR code. We have exploited this particular feature of the AR technology to embed unique and innovative educational affordances in our learning activity.

3.3. *Description of the activity*

Four 15-year-old Swedish students, from the same class, were recruited to participate in the study. Alice, Ralph, Edward and Larry (the names of the students have been changed) were used to working together in school subjects other than mathematics. In addition, we were informed that they had already covered the concept of scale in school.

When the students arrived to the classroom they could see a satellite photograph of their hometown, measuring 70 cm x 100 cm and marked with the scale 1:800. The photograph was taped down on a table, that was placed 1.5 meters directly in front of the whiteboard showing a projected image of the photograph. The session started with a short presentation of how the equipment worked. The students were asked to make the webcam read the first tag and see what happens. They could then see the building (the Turning Torso) on the whiteboard and got excited when they figured out that they could move and turn the building on the whiteboard by just changing the position of the tag on the table.

The mathematical task was communicated through written instructions, including information on how the tags worked. The students were simply asked to figure out the building's height in reality. They were free to use tools and take measurements both on the whiteboard and on the table, but this was not explicitly communicated. A whiteboard pen, magnetic dots, a ruler and a calculator were readily available for them to use. The students were informed that they could consult their temporary teacher (the author of this paper) during the activity, if they had any questions.

The activity involves no step-by-step instructions or suggestions on how to proceed. Instead, the students have to rely on their mathematical pre-knowledge and collaborative self-regulatory skills, combined with guidance provided by the teacher. Although the activity is not structured, all hypothetical learning trajectories will have to involve one of two possible strategies. These two strategies will be discussed in the next section.

3.4. *Prospective analysis of the activity*

We will now account for the mathematical concepts, methods and strategies that the students may bring to use for interpreting the building's height in reality. The activity involves three distinct referential contexts (Figure 2: table, whiteboard, reality) that are intended to stimulate the learners' reasoning and support their decision-making.

We proceed to account for two main strategies for completing the task. The first strategy, which we refer to as the comparison strategy, has two substrategies which both depart from measuring the building on the whiteboard (b) by using the ruler. Unfortunately, the scale 1:800 that is indicated on the photograph on the table (a) does not directly apply to the height measured on the whiteboard (b). Instead, this measured height may be compared with corresponding distances measured horizontally (to ensure the same scale) on the whiteboard (b). This can be done in at least two different ways.

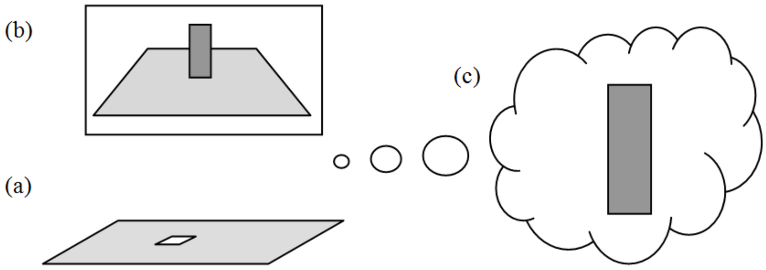

Figure 2. (a) The table context, (b) the whiteboard context, (c) the reality context.

The first comparison substrategy involves identifying two points on the projected photograph, both located horizontally from the projected tag, so that the distance between the two points equals the measured height of the building (Figure 3, left pane). These two points are identified on the whiteboard. In order to apply the scale, the corresponding points on the table have to be identified so that the distance between them can be measured. For this measurement on the table the scale 1:800 applies and the task may be completed by multiplying the table measurement with 800 and adjusting the length unit from centimeter to meter.

The second comparison substrategy involves identifying an object on the projected photograph that is located horizontally from the projected tag, for example a car, and measure its length. The tag may be moved next to a position where an object of known length is available. Measuring the object and the building on the whiteboard provides two measurements. The corresponding measurement of the chosen object in reality is supposed to be known or may be estimated. We may now proceed either by computing a new scale (object in reality against object on the whiteboard) and apply it to the measured height of the building on the whiteboard, or applying principles of similarity (proportions, Regula de Tri) in arithmetic form or algebraically, by solving an equation.

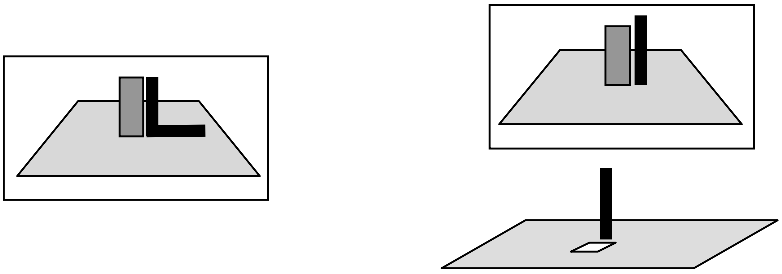

Figure 3. The comparison strategy (left pane) and the coordination strategy (right pane).

The coordination strategy (Figure 3, right pane) requires identifying that the technical set-up affords projecting physical objects onto the whiteboard by placing them on the table. The identification of this affordance is stimulated by moving tags around on the table and seeing that the hand moving the tag around is also projected onto the whiteboard. This affordance may be exploited by placing an object, preferably the ruler, on the table (a) and next to the paper tag. The projected ruler shows up next to the building on the whiteboard where the height (in centimeters) may be read off on the projected ruler. In this case, the measurement in centimeters refers to the table context and the scale 1:800 may be applied to find the corresponding measure in reality.

This prospective analysis has influenced the design process and has specifically underpinned the creation of embedded scaffolds and critical scaffolding actions provided by the teacher during the implementation of the activity.

3.5. *Embedding affordances for a critical scaffolding action*

The availability of a building on the whiteboard, a scale on the table, and a ruler, invites the students to measure the building on the whiteboard and apply the scale to the measurement. However, this strategy results in an incorrect answer, since the scale on the table does not apply to the measurement on the whiteboard. This feature of the activity challenges the students' self-regulation, particularly regarding reaction and reflection. If they do not realize by themselves that the strategy is incorrect, a scaffolding action is needed to make them realize that they need to change strategy.

This need for appropriate scaffolding was addressed by embedding affordances for creating a cognitive conflict. We introduced a technical requirement that the size of the image of a projected object should change by at least a factor of 2 from front to back, so that the image is twice as large when the object is placed close to the webcam as compared to being placed far away from the webcam, but still on the photograph.

This size change allows the students to obtain two different measurements for the projected object by measuring the image using a physical ruler on the whiteboard: first measuring the (image of the) object when it is placed close to the webcam (Figure 4, right

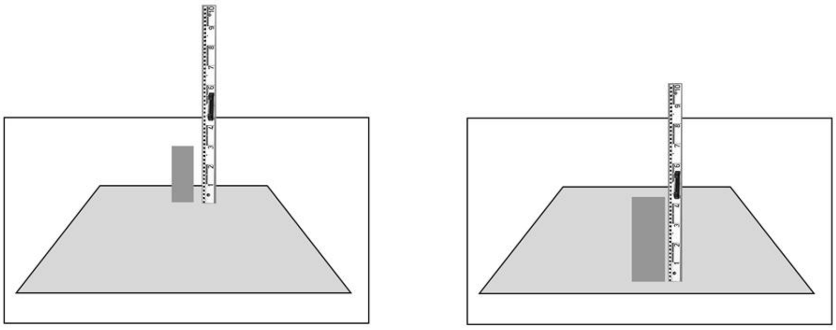

Figure 4. The measurements differ on the physical ruler.

pane) and then measuring it again when it is placed farther away (Figure 4, left pane).

Since the measurements relate to the same object, they should be the same. At this stage, the students should realize that something must be wrong and thus we have achieved a cognitive conflict, affording the students to reflect on their strategy and stimulate them to engage in renewed forethought, revise their planning and activate a modified strategy.

The teacher should be prepared to stimulate the students to experience this cognitive conflict, if needed, by intervening in their inquiry and performing a scaffolding action for example by asking them to measure the building at different locations. The students may then realize by themselves that it is not reasonable to obtain different measurements for the same object, or they may be asked further scaffolding questions such as “Does the height of the building change when it is moved around?”. Of course, in a virtual world it is certainly possible to avoid the rules of reality and have a building changing heights (cf. Sutherland, 1965). However, the current task concerns an object that the students should recognize from reality, namely the Turning Torso, as a building having a well-defined height that is not supposed to change as it is moved around.

If the students manage to continue their inquiry and possibly apply the previously described coordination strategy, they may confirm this strategy by moving the ruler on the table and next to the tag in order to confirm that the measurement does not change as the tag is moved on the table (Figure 5).

4. Results and Analysis

The four selected students were all 15 years old and used to working together in other school subjects than mathematics. They volunteered when the teacher asked the class who would like to participate in solving a mathematics problem with new technologies. The teacher told us that the students knew how to do calculations with scales, but was not so sure if they would be able to apply that knowledge in a problem solving situation.

The students spent 30 minutes working on the task. The session was videotaped and additionally audio recorded. During the activity, it was mainly Alice and Ralph that took initiatives in the group work. Larry was more or less quiet during the whole session.

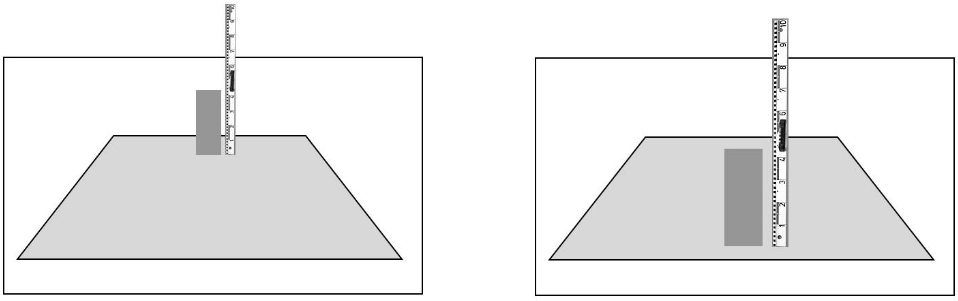

Figure 5. The measurement remains the same on the projected ruler.

Edward contributed by moving the tags, mostly on Alice or Ralph's initiatives.

We will account for two specific episodes that occurred in this session. The first episode illustrates how the students engage in collaborative self-regulatory processes, while the second episode highlights how guidance and scaffolding actions stimulate their continued engagement in the inquiry.

4.1. *Students' collaborative self-regulation of the inquiry*

Alice takes the first initiative and makes the group aware that the scale on the photograph says 1:800. She points to the whiteboard and asks the group about the height of the image. On Ralph's suggestion, Alice takes a ruler, 50 cm in length, and starts to measure the height of the building on the whiteboard.

Alice: [Measuring the height]…well…
Ralph: 39?
Alice: Yes, 39.
Ralph: [writing down 39 on a paper] Multiply with 800 then?
Group: Yes.

The interaction between Alice and Ralph, together with the confirmation from the whole group, can be interpreted as an instance of collaborative control. The students know how to apply a scale to a measurement of an image, but instead of reflecting individually on the chosen strategy they choose to exert control by asking their peers.

Next, Ralph asks for a calculator. However, while he waits, he finishes the calculation with a paper and pencil algorithm.

Ralph: …then it will be….
Alice: 31200.
Ralph: It is not that high!
Alice: Is it not?
Ralph: No, it can't be 3000 meter high.
Alice: [Laugh] think about an airplane…
Edward: Yes, but it is centimeters.

The group comes to an agreement of 312 m for the height of the building. Their reasoning, emerging in the frame of augmented reality, is coordinated and tested against what is known and seems to be plausible in the real world. This testing gives incitement for collaborative reflection, which leads them to adjusting their preliminary answer.

4.2. *Scaffolding actions promoting continued inquiry*

The students do not have access to the exact height of the building, but the estimation appears plausible for them. The teacher is informed about the prospective analysis and thus knows that the students have applied a mathematically incorrect strategy that has resulted in an incorrect answer. They ask the teacher if their answer is correct. The teacher activates the prepared scaffolding action, asking them “What if you try to measure the building at different places?” As the students do not seem eager to engage in such actions, maybe because they expected an explicit yes-or-no answer, he performs the scaffolding action himself while discussing with the students.

Teacher: We can test something here. If we look at it here [drags the tag away from the webcam] and then place it here [drags the tag closer to the webcam], then it [the 3D image] becomes larger.

Ralph: Yes, then it will not be the same.

Teacher: Isn’t that strange? Then it will be higher when you measure on the board, or? [moving the tag away from the camera so that the size of the building is reduced on the board].

When the teacher moves the tag it becomes obvious to the students and particularly to Ralph that there are flaws in their reasoning. They exert control by abandoning their first strategy when realizing that their answer depends on how the tag is positioned in relation to the webcam, and engage in the exploration of new strategies.

Alice suggests that they should have something on the table to compare the building with. Edward, who has been noting that the building may be laid down on the table if the tag is tilted towards the webcam, asks if this feature of the technology could be used. The group makes a short effort to explore his initiative. In doing this, they agree on that the picture of the building has to appear horizontally on the whiteboard in order to provide an appropriate measure to use for calculations. However, when Edward tries to place the building horizontally, the tag loses contact with the webcam and the building disappears.

After a moment of silence Alice continues:

Alice: But, if one tries to measure something here on the map and then compares that to…

Ralph: Yes, but, there is nothing on the map that is marked down, it is just… it is 3D.

While Alice suggests making use of a physical object on the table, Ralph proposes to make use of a virtual object. When that does not work out, he seems unwilling to engage in the strategy suggested by Alice. By engaging in personal planning processes, their collaboration turns into an unproductive competition and conflicting views between the two students.

They do not know how to proceed, remain silent for about 7 seconds and seem discouraged. The teacher chooses to intervene and shows the group how the webcam can project the ruler in an upright position on the board, attempting to make them discover the previously described coordination strategy. On the table only the ruler can be seen, but on the board the ruler is shown together with the building.

On account of this intervention, Ralph asks Edward to drag the tag to different positions on the table. Ralph follows Edward's moves and places the ruler beside the tag. The group discovers that the building's height always stays at 24 centimeters on the ruler, regardless of where the comparison is made. Now the ruler is used in the table context, while the students' reasoning requires that the table context is coordinated with the whiteboard context where the measurement 24 cm may be read off. Ralph calculates the new height to 192 meters by multiplying 24 with 800 and transforming to meters. The height of 192 meters seems plausible to the students and the teacher also confirms this by acknowledging their solution (the exact height of the Turning Torso is 190.4 meters). The group continues to measure some of the other buildings, by applying the same strategy.

After the students leave the room, their teacher comments that they should have the opportunity to engage more often in this kind of activities to get used to working on own initiatives and develop confidence to explore different strategies.

5. Discussion

We have analyzed the design process and the learning outcomes of a collaboratively designed mathematical inquiry involving augmented reality. The AR technology made it possible to create a rich learning environment with several referential contexts and embedded scaffolding features that supported and stimulated the learners' mathematical inquiry, specifically their self-regulatory processes, as well as the teacher's scaffolding actions. In the design process, we focused more on enhancing the mathematical learning outcomes of the activity than fully exploring the affordances of the AR technology. However, the AR technology, as one among many available technologies and traditional resources, served us well in facilitating the design of an innovative mathematical inquiry with unique embedded learning opportunities.

5.1. *Designing for inquiry and guidance*

The activity involved three referential contexts: table, whiteboard, and reality. If AR had not been used, and instead real models of buildings had been placed on the table, the task of determining the height of the building could have been solved by just involving the table context. Such a task could possibly be solved as a routine exercise and would not necessarily challenge the learners to engage in an inquiry involving constructive mathematical processes. The set-up made possible with the AR technology required the learners to coordinate at least two of the three referential contexts (table-whiteboard, whiteboard-reality, or all three) in order to successfully complete the inquiry. The activity challenged them to engage in collaborative self-regulatory processes such as forethought, planning, activation, monitoring, control, and reflection. With some guidance, they identified and coordinated the affordances needed to efficiently represent the task, conjecture, solve the task, and justify their answer.

The prospective analysis, which focused on constructing hypothetical learning trajectories involving artifacts and their affordances, served to identify critical aspects of the activity and support the design of corresponding scaffolding actions. These actions were carefully prepared so that they would stimulate the learners' continued engagement in self-regulatory processes, as compared to momentary unprepared actions that are often limited to explicit yes-or-no answers. As the outcomes show, the particular scaffolding action involving the creation of a cognitive conflict played a crucial role in stimulating the learners' continued inquiry.

5.2. *Rich learning environments supporting inquiry*

The three referential contexts provided a variety of affordances for the students' self-regulatory processes. The reality context was enhanced by a photograph of the center of their home city center and the involvement of familiar buildings. This context was used and augmented by the students' personal experiences (for example, how high an airplane flies) as a context for reflection and for controlling their collaborative reasoning.

For the learners, the AR technology appeared in the classroom environment simply through the movable paper tags that afforded moving virtual objects on the whiteboard. The guidance provided by written instructions and by the teacher served to support the students to identify the available contextual affordances that they could use as references in their mathematical inquiry. Although the comprehensive learning activity could appear as quite complex from a teacher perspective, it appeared much less complex from a user perspective allowing the learners to readily engage in their mathematical inquiry.

On one particular occasion, we chose to exclude a referential setting that may have proved useful in further supporting self-regulation. As accounted for in the second episode, the teacher simply acknowledges the students' solution 192 meter when the students could have checked the answer themselves, for example by searching on the Internet or by further reasoning. Although such a solution was considered in the design process, it was decided by the design team not to involve additional computers in order to reduce the technical complexity of the comprehensive set-up. We decided it was better to keep a simple set-up in order not to impose too much technology on the students, particularly since they were not used to engaging in explorative activities.

An essential aspect of DBR is to decide what contextual affordances should be embedded in the learning environment. Such affordances serve to support the students' learning trajectories and self-regulatory processes as well as the teacher's scaffolding actions. The learning environment should preferably offer a variety of contexts as well as opportunities to vary and coordinate affordances within and across contexts.

5.3. *Learners' experiences and learning gains*

During the session it was apparent that the learners were focused on their inquiry, involving several cognitive challenges and learning opportunities. They directed their full attention towards successfully completing the inquiry. The provided guidance was minimized in the sense that the learners were encouraged to pursue their collaborative inquiry on their own, and were given ample time to do so. Explicit guidance was provided only when absolutely needed, for example when the learners were discouraged and could not motivate themselves to continue. In this respect, the presented results are not representative of the comprehensive inquiry as they emphasize the few critical situations that appear during the session. The session was dominated by the learners' productive and constructive collaborative discussions, that they pursued with interest and joy.

Regarding learning gains, we did not evaluate the learning outcomes. Instead, we chose to focus on designing for inquiry and to rely on the scientifically well established implication that engaging in interactive-constructive processes, where the learners have agency and authority, result in significant learning gains (Chi, 2011; Boaler, 2012). We strongly believe that educational research should put more focus on case-studies where the designed and carefully situated inquiries are evaluated against the processes they support rather than evaluating learning gains that can readily be predicted by referring to previous research. However, relying on the effects of processes implies that these processes have to be stimulated and pursued by the learners. For this reason, we provided the guidance needed for the learners to continue and complete their inquiry, to ensure that they would achieve a full learning experience.

6. Concluding Remarks

In the current paper, we have provided an example of an innovative mathematical inquiry that invites learners to collaborate in exploring, analyzing and coordinating a variety of contextual affordances when solving a task related to the concept of scale, thus stimulating them to engage in goal-oriented self-regulatory and mathematical processes.

Our inquiry is supported by the AR technology, which has served our purposes well by providing unique referential contexts that cannot be made available in a traditional learning environment. Other design efforts may benefit from using other technologies. However, a technology centric approach is not sufficient for advancing the use of technologies in education. The challenge posed by Kaput (1992) to exploit technological affordances for learning needs to be further addressed by pursuing learner centric research and development involving technologies that are readily available on laptops, interactive whiteboards, and mobile devices (cf. Chen, Lee, Tan, Wettasinghe, & Wong, 2012).

The video see-through approach mediated by inexpensive paper tags is a reasonable strategy for implementing AR in education. As noted earlier, even this approach requires substantial design efforts regarding programming and didactical engineering and is not readily implemented by individual school teachers. Furthermore, copyright issues and associated costs for making use of available three-dimensional pictures has a negative effect on the provision of creative and stimulating AR applications for educational purposes. It may well be the case that making use of tangible paper or physical object interfaces that replace expensive equipment, exemplified by Dor Abrahamson's embodied design approach (2011), has far greater educational potential than the video see-through version of the AR technology.

In future research efforts involving tangible interfaces, we will apply a broader approach inspired by the notion of embodied design. It may happen that we again end up making use of the AR technology, but then as a consequence of working in a broader technological context while still prioritizing provision of opportunities for enhancing children's mathematical learning experiences.

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