

HOW I VIEW LEARNING AND THINKING IN CSCL GROUPS

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The field of CSCL is particularly interested in the ways small groups can build knowledge together thanks to communication and support from networking technology. I hope that CSCL environments can be designed that make possible and encourage groups to think and learn collaboratively. In my research, my colleagues and I look at logs of student groups chatting and drawing about mathematics in order to see how they build on each other's utterances to achieve more than they would individually. To answer this important question, we must look carefully at the details of discourse in CSCL groups and develop innovative tools and theories. In this talk¹, I outline methods and levels of analysis that have resulted in the findings reported in the Virtual Math Teams research cited in the references.

Keywords: CSCL; interaction analysis; group cognition.

1. Views of Learning and Thinking

Learning without thought is labor lost; thought without learning is perilous.
(Confucius)

About 2,600 years ago, Confucius viewed *learning* and *thinking* as belonging together.

The learning sciences of the twenty-first century agree. They view learning as involving meaning making by the learners (Stahl *et al.*, 2006). Students who just passively accept instruction without thinking about it and coming to understand it in their own way of making sense of things will be wasting everyone's time. Why? Because they will not be able to *use* the new knowledge or to *explain* it. Of course, someone can learn

¹ This paper is based on an invited keynote talk given at the 17th International Conference on Computers and Education (ICCE 2009) on November 30, 2009, in Hong Kong, China (see video at <http://www.youtube.com/user/GerryStahl#p/u/2/h5MpUJnTipM>). The intent of the original talk was to provide a personal view of the field of computer-supported collaborative learning (CSCL) and to relate it to the Asia-Pacific audience. In the talk the author tried to describe—in an informal tone—his approach to analyzing online interaction in small groups. The author has tried to retain the original tone in publishing the talk.

something one day and make sense of it later, when they try to use it in different circumstances and to explain their use to other people and to themselves. But if they never integrate what they have learned into their own thinking and acting—by applying it where appropriate and talking about it clearly—then they will not have learned anything important.

What we, as learning scientists, have learned in recent decades in the West is influenced by what philosophers before us said (Stahl, 2011b). For instance, most Western philosophers until the middle of the 1900s thought that knowledge could be expressed by propositions, sentences or explicit statements. If that is true, then the learning of knowledge can consist simply of students hearing or reading the right sentences and remembering them.

But Ludwig Wittgenstein's book, *Philosophical Investigations*, published in 1953, questioned this view of learning and thinking. It looked at math as a prime example. Mathematical knowledge can be seen as a set of procedures, algorithms or rules. Wittgenstein asked how one can learn to follow a mathematical rule (Wittgenstein, 1944/1956, 1953). For instance, if someone shows you how to count by fours by saying, "4, 8, 12, 16," how do you know how to go on? Is there a rule for applying the rule of counting by fours? (Such as, "Take the last number and add 4 to it.") And if so, how do you learn that rule? By another rule? Eventually, you need to know how to do something that is not based on following a propositional rule—like counting and naming numbers and recognizing what numbers are larger. The use of explicit rules must be somehow grounded in other kinds of knowledge. These other kinds include the tacit knowledge of how to behave as a human being in our culture: how to speak, count, ask questions, generalize, put different ideas together, apply knowledge from one situation in another context, and so on. Wittgenstein's question brought the logical view of knowledge as explicit propositions into a paradox: if knowledge involves knowing rules, then it must involve knowing how to use rules, which is itself *not* a rule.

Wittgenstein was an unusual philosopher because he said that problems like this one cannot be solved by contemplation, but rather by looking at how people actually do things. He said, "Don't think, look!" I try to follow Wittgenstein's advice. I try to view how people actually *do* things. Rather than telling you what my *views* or ideas are about learning and thinking in CSCL groups, I want to *show* you how I *view* or observe learning and thinking in CSCL groups.

The term "view" in the title of my talk has this double meaning: it means both viewing by looking at something with my eyes and also viewing in the metaphorical sense of thinking about something from a conceptual perspective. The Greek philosopher, Plato, who lived at about the same time as Confucius, made this metaphor popular in Western thought (Plato, 340 BC/1941).

Although Wittgenstein himself did not actually look at empirical examples of how people follow rules in math, we can. By carefully setting up a CSCL session, we can produce data that allows us to view small groups of students learning how to follow math

rules and thinking about the math rules. This is what I do to view learning and thinking in CSCL groups. It is the basic approach of the science of group cognition (Stahl, 2009b) that I want to describe today.

The work of my research team and other colleagues involves looking closely at some rich examples of student groups learning and thinking about math. I would like to share a brief excerpt from one of these examples with you and talk about how we go about viewing the learning and thinking in this group of students.

2. An Example of Learning and Thinking

2.1. The event: VMT Spring Fest 2006 Team B

Here, I will be talking about an online event that occurred several years ago. Three students, probably about 16 years old, met with a facilitator in an online chat environment on May 9, 10, 16, and 18, in 2006, for about an hour in the late afternoon each day. The participants were distributed across three time zones in the US. The event was part of the Virtual Math Teams (VMT) research project of the online Math Forum (Stahl, 2009a).

The topic for this event was to explore a pattern of sticks forming a stair-step arrangement of squares (see Figure 1) and then to explore similar patterns chosen by the students themselves. The VMT online environment consisted primarily of a synchronous chat window and a shared whiteboard. At the end of each session, the students were supposed to post their findings on a wiki, shared with other teams participating in the

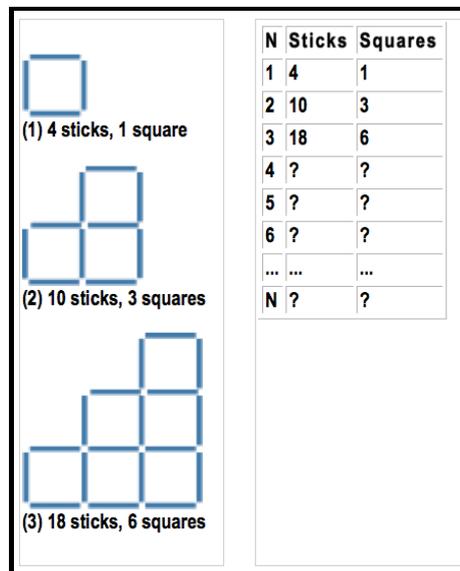


Figure 1. Topic for VMT Spring Fest 2006.

Spring Fest. Between sessions, the facilitator posted feedback to the students in a textbox on the whiteboard.

2.2. The session: Session 3, May 16, 7 pm

I want to look at an excerpt from the end of the third session. The three students had already solved the original problem of the stair-step pattern of squares. They had also made up their own problem involving three-dimensional pyramids. Now they turned to look at the problem that Team C had described on the wiki after session 2. Team B is looking at an algebraic expression that the other team of students had derived for a diamond pattern of squares. They start to draw the pattern in their whiteboard (see Figure 2) and chat as a team about the problem of this new pattern.

2.3. The theme: “I have an interesting way to look at this problem”

One of the students, Aznx, makes a proposal on how to “look” at their problem. First, he announces that he has an interesting way to look at it. Note that he uses the word “look” in the same double meaning that I do in my talk title. As we will see, he means he has a new way to think about the problem mathematically—and that involves a way of observing a visual image of the problem. The group does its thinking both by typing text

The screenshot displays the VMT Replayer interface. The main window shows a whiteboard with several diagrams and text:

- A diagram of a 3D pyramid-like structure made of blocks, with a label "top/bottom: $2n(n+1)$ ".
- A diagram of a diamond pattern of squares, with a label "Derived from $\frac{(1+n)^2 + n^2 + n^2 + n^2}{2}$ ".
- A diagram of a 2D pattern of squares, with a label " $2n(n+1) + (n+1)^2$ ".
- Handwritten notes: "3rd STEP", "2nd STEP", and "4th STEP".
- A diagram of a diamond pattern of squares, with a label " $2n(n+1) + (n+1)^2$ ".

The chat window on the right shows the following messages:

```

Aznx:
Gany:
QuickSilver:
bwang8

bwang8 5/16/06 7:33:00 PM EDT: OK
bwang8 5/16/06 7:33:16 PM EDT: lets think about the equatin
bwang8 5/16/06 7:33:22 PM EDT: equation
QuickSilver 5/16/06 7:33:22 PM EDT: yes
bwang8 5/16/06 7:33:29 PM EDT: how did they derive it
Aznx 5/16/06 7:33:50 PM EDT: There's the formula
bwang8 5/16/06 7:34:52 PM EDT:  $(n+2) + (n-1)^2 + 2 = n^2 + 3 - 2$ 
bwang8 5/16/06 7:34:58 PM EDT:  $n^2 + 2 + 4(n-1) + 2$ 
Aznx 5/16/06 7:34:58 PM EDT: the 3n has to do with the growing outer layer of the pattern I think.
QuickSilver 5/16/06 7:34:25 PM EDT: the sides and squares
Aznx 5/16/06 7:34:55 PM EDT: right.
Aznx 5/16/06 7:35:00 PM EDT: There.
Aznx 5/16/06 7:35:26 PM EDT: I have an interesting way to look at this problem.
QuickSilver 5/16/06 7:35:42 PM EDT: Tell us
Aznx 5/16/06 7:35:46 PM EDT: Can you see how it fits inside a square?
bwang8 5/16/06 7:35:49 PM EDT: yes
QuickSilver 5/16/06 7:35:52 PM EDT: Yes
bwang8 5/16/06 7:35:53 PM EDT: oh
bwang8 5/16/06 7:35:55 PM EDT: yes
QuickSilver 5/16/06 7:36:02 PM EDT: You are sayin'g extra spaces...
Aznx 5/16/06 7:36:05 PM EDT: Also, do you see if you add up the missing areas
QuickSilver 5/16/06 7:36:11 PM EDT: Yes...
QuickSilver 5/16/06 7:36:18 PM EDT: they look similar to the original figures
QuickSilver 5/16/06 7:36:21 PM EDT: figure
Aznx 5/16/06 7:36:22 PM EDT: It is equivalent in size to the small circle in the pattern
QuickSilver 5/16/06 7:36:24 PM EDT: Small circle?
Aznx 5/16/06 7:36:39 PM EDT: The only part you would be missing out are the four squares
Aznx 5/16/06 7:36:49 PM EDT: on the outer areas of this square
Aznx 5/16/06 7:37:00 PM EDT: Did you guys get what I mean?
bwang8 5/16/06 7:37:02 PM EDT: yes
QuickSilver 5/16/06 7:37:08 PM EDT: Show what u mean on the whiteboard
QuickSilver 5/16/06 7:37:11 PM EDT: I dont get it
Aznx 5/16/06 7:37:14 PM EDT: bwang8 you show him
Aznx 5/16/06 7:37:17 PM EDT: since you get it
  
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At the bottom of the interface, there is a speed control and a status bar showing "Aznx is typing" and "Current action at: 7:37:17 PM (Message By: Aznx)".

Figure 2. The VMT Replayer showing the VMT online environment.

and algebraic expressions in the chat window and by simultaneously drawing and viewing diagrams or geometric constructions of the problem in the shared whiteboard (Çakır *et al.*, 2009).

Azxn's announcement opens an opportunity for the group to discuss a way of looking at the problem that Azxn would like to share with the group. In fact, the group takes up Azxn's offer and the students spend the next eight minutes trying to each understand it. As it turns out, they will work on Azxn's view of the problem for the rest of this session and most of their final session.

A VMT chat session can generally be analyzed as a series of themes or discussion topics. Often, themes come and go, and different themes overlap, with one wrapping up while another starts up. Researchers can identify the boundaries of a theme: when a new theme opens and an old one closes (Zemel *et al.*, 2009).

In this case, the group has been talking about how the diamond pattern grows as a geometric figure for a couple of minutes and then they discuss Team C's algebraic expression for a couple of minutes. As those themes get played out and there is a pause in the chat, Azxn makes a move to open a new theme for the group.

2.4. A move: Showing how to view the problem

Azxn's announcement that he has an idea to share with the group is a way of introducing a new theme. Conversations often flow by new contributions picking up on something that was already being discussed. Online text chat tends to be more open than face-to-face talking; chat does not follow the strict turn-taking rules of conversation. However, it is still common to do some extra work to change themes even in chat. In a sense, Azxn is asking permission from the group to start a new theme. Quicksilver responds encouragingly right away by saying, "Tell us" (see Figure 3).

line	date	start	post	delay		
919	5/16/06	19:35:26	19:35:36	0:00:06	Azxn	I have an interesting way to look at this problem.
920	5/16/06	19:35:41	19:35:42	0:00:03	Quicksilver	Tell us
921	5/16/06	19:35:38	19:35:45	0:00:00	Azxn	Can you see how it fits inside a square?
922	5/16/06	19:35:45	19:35:45	0:00:07	Bwang	yes
	5/16/06	19:35:49	19:35:52	0:00:00	Bwang	
923	5/16/06	19:35:51	19:35:52	0:00:01	Quicksilver	Yes
924	5/16/06	19:35:52	19:35:53	0:00:02	Bwang	oh
925	5/16/06	19:35:55	19:35:55	0:00:06	Bwang	yes
926	5/16/06	19:35:53	19:36:01	0:00:04	Quicksilver	You are saying the extra spaces...
927	5/16/06	19:35:58	19:36:05	0:00:06	Azxn	Also, do you see if you add up the missing areas

Figure 3. The move to introduce Azxn's new way of looking at the group's problem. (This log for analysis encodes the chat stream and associated awareness messages about when people started typing, along with timing data to reflect the flow of discourse.)

Actually, Aznx already starts typing his proposal before he gets Quicksilver's response, but it is not posted until afterward. His proposal is: "Can you see how it fits inside a square?" Here, he structures his proposal as a question, in order to elicit a response from the other members of the team. Note that he uses the term "see" in his proposal with the same double meaning as the term "view" in his prior announcement. As we shall see (in both senses), the group tries to work out and comprehend Aznx's proposal both conceptually and visually.

Both Bwang and Quicksilver respond to Aznx's proposal with "Yes". However, both modify this response. Bwang starts to type something else, but erases it; then he posts two messages: "oh" and "yes". This indicates that he did not immediately understand the proposal fully, but it took him a couple of steps over a few seconds. Quicksilver follows his initial positive response with, "You are saying the extra spaces ..." He is looking for more clarification of the proposal. While Quicksilver is typing his request for clarification, Aznx is typing an expansion of his initial proposal: "Also, do you see if you add up the missing areas ..."

The analysis of interaction moves is central to the science of group cognition (Stahl, 2006a). This is the level of granularity of many typical group-cognitive actions. In ethnomethodology, these are sometimes called "member methods" (Garfinkel, 1967). They are ways in which small online groups get their work done. They often follow conventional patterns, which makes them much easier for participants to understand. Researchers can also look for these patterns to help them understand the interaction taking place by the group.

In this case, the action taking place is the opening of a new theme, one which will provide direction for the rest of this group's event together. This move is an example of one way in which a group can establish a shared understanding of a diagram or select a joint problem conceptualization (depending on how we understand the terms "look" and "see"). Other moves that we often see in VMT logs are defining shared references, coordinating problem-solving efforts, planning, deducing, designing, describing, solving, explaining, defining, generalizing, representing, remembering, and reflecting as a group.

2.5. A pair: Question/response: "Can you see how it fits inside a square?" / "Yes"

In conversation analysis, one typically looks for "adjacency pairs" (Duranti, 1998; Sacks, 1962/1995). A prototypical adjacency pair is question/answer. Aznx's offering of a question—"Can you see how it fits inside a square?"—followed by Bwang and Quicksilver's response—"yes", "Yes"—illustrates this structure for the simplest case: one person poses a yes/no question and the others respond with an affirmative answer.

Response structures are often more complicated than this. Text chat differs from talk in that people can be typing comments at the same time, they do not have to take turns and wait until one person stops talking and relinquishes the floor. They will not miss what the other person is saying, because unlike with talk, the message remains observable for a while. The disadvantage is that one does not observe how people put together their

messages, with pauses, restarts, corrections, visual cues, intonations, and personal characteristics. While it is possible to wait when you see a message that someone else is typing, people often type simultaneously, so that the two normal parts of an adjacency pair may be separated by other postings. For example, Quicksilver's question (line 926 in Figure 3) separated Aznx's continuation of his line 921 posting in line 927, because 926 appeared before 927 although 927 was typed without seeing 926. So in chat we call these "response pairs" rather than "adjacency pairs." While they may be less sequentially *adjacent* than in talk, they are still direct *responses* of one posting to another.

Because the sequencing in online chat texting is less tightly constrained than in face-to-face talk, response pairs are likely to become entangled in the longer sequences of group moves. This may result in the common problem of "chat confusion" (Fuks *et al.*, 2006; Herring, 2001). It can also complicate the job of the researcher. In particular, it makes the task of automated analysis more complicated. In convoluted chat logs, it is essential to work out the response structure (threading) before trying to determine the meaning making. The meaning making still involves participants interacting through the construction of response pairs, but in chat people have to recreate the ties among these pairs. Realizing this, the group members design their postings to be read in ways that make the response pair or threading structure apparent, as we will see (Zemel & Çakir, 2009).

2.6. An utterance: Question: "Can you see how it fits inside a square?"

In his posting—"Can you see how it fits inside a square?"—Aznx is comparing the relatively complicated diamond shape to a simple square. This is a nice strategy for solving the group's problem. The group can easily compute the number of stick squares that fill a large square area. For instance if there are five little squares across the width of a square area (and therefore five along the height), then there will be five-squared, or 25 little squares in the area. In general, if there are N little squares across the width, there will be N -squared to fill the area. This is a strategy of reducing the core problem to a simple or already known situation—and then perhaps having to account for some differences. So Aznx's posting seems to be relevant to thinking about the math problem conceptually.

At the same time, Aznx poses his proposal in visual or graphical terms as one of "seeing" how one shape "fits inside" of the other. The group has been looking at diagrams of squares in different patterns, both a drawing by Team C in their wiki posting and Team B's own drawings in their whiteboard. So Aznx seems to be visualizing a possible modification to one of the diamond drawings, enclosing it in a square figure (see the blue diamond pattern enclosed in the red square in Figure 4). He is asking the others if they can visualize this also, so that the group can use this to simplify and solve their problem with the diamond.

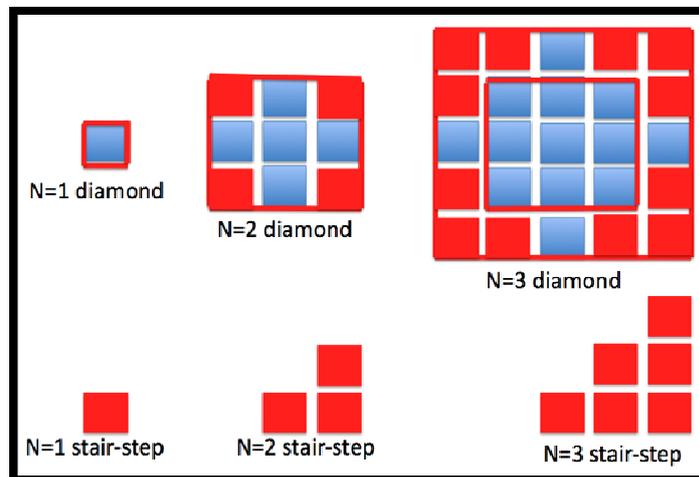


Figure 4. Blue diamond patterns and red stair-step patterns.

Azxn presents his proposal about re-thinking the problem as a question about visualizing the diagram. The group has been working in the VMT environment, going back and forth between text in the chat, and drawings in the whiteboard. They have started with problems presented graphically and have discussed these graphical problems in their text chat. They have shared different ways of viewing the relationships within the drawings and they have gradually developed symbolic algebraic ways of expressing general relationships about patterns in these drawings, working out these symbolic expressions in the chat and then storing them more persistently in the whiteboard.

I have been calling Azxn's chat posting a "problem-solving math proposal" (Stahl, 2006a). However, it is presented in the grammatical form of a *question*. Azxn does not simply state a proposal like, "I think we should enclose the diamond in a square, calculate the size of the square and then subtract the missing areas." Rather, he first announces that he had "an interesting way to look at this problem" and then explains his way of looking by asking if the others could "see how it fits inside a square." Presenting a proposal calls on the others to accept the proposal and start to work on it. Of course, the others can reject the proposal, ask for clarifications about it, make a counter-proposal or ignore the proposal. But Azxn's utterance avoids formulating a full proposal, which the others could then accept or reject. Rather, it presents a preliminary step. It asks the others if they can visualize something. It puts this to them as a question. If they say yes, then Azxn can proceed to make his proposal—or perhaps the others will see the implications of his interesting way to look at the problem and propose the strategy without Azxn having to advocate it, explain it, and defend it. If they say no—that they cannot see how it fits inside a square—then he can explain his view further so they will be better prepared to accept his proposal. In this way, Azxn delays indefinitely articulating a complete

proposal; by starting the conversation about the visualization he will be able to involve the others in articulating the proposal *collaboratively*. In fact, in the subsequent discussion, the others do “see” the strategy that is implicit in Aznx’s interesting view of the problem and they do help to articulate the strategy and then pursue it. By designing his proposal as this preliminary question about viewing the problem, Aznx succeeds in directing the group problem-solving in a certain direction without his having to fully work out a detailed, explicit proposal.

2.7. A reference: “It”

Aznx’s question is ambiguous at a purely syntactic level. He asks the others, “Can you see how *it* fits inside a square?” What does the term “it” refer to? People use pronouns like “it” rather than lengthy explicit noun phrases when the reference is clear from the context. This makes the utterance situated in its context—it’s meaning cannot be gathered from the utterance considered in isolation. Often, “it” will reference something that was recently referred to in a previous contribution that the new utterance is building on. For instance, “it” could refer to something mentioned in Aznx’s previous utterance, “I have an interesting way to look at this problem.” But to say that it refers to “this problem” does not make complete sense. The *problem* does not fit inside a square.

However, a minute earlier, when the group was discussing Team C’s equations, Aznx said about part of an equation, “The 3n has to do with the growing outer layer of the pattern I think.” He was referencing different aspects of the growth of the diamond pattern, particularly its “outer layer.” So when he announces that he has an interesting way to view the problem, it is reasonable to assume that his new way of looking may be closely related to the observation that he had just reported about the outer layer of the diamond pattern. Because everyone in the group was following the flow of the discussion, Aznx could refer to the topic of the outer layer of the diamond pattern in the shorthand of the pronoun “it”. When he typed, “Can you see how it fits inside a square?” he could assume that the readers of this posting would understand that he was referring to how some aspect of the diamond pattern can be seen as fitting inside of some square shape.

Although the reference to some aspect of the diamond pattern is relatively clear, the details are not clear about just what aspect of the diamond is to be visualized or focused on visually, where a square is to be constructed, and how the diamond fits inside the square. At this point, only a rather confusing image of a diamond pattern is visible on the whiteboard (see Figure 2).

Bwang and Quicksilver both respond initially to Aznx’s question with “Yes.” However, as we saw, Bwang indicates some hesitancy in his response and Quicksilver asks for further clarification. Aznx and Quicksilver discuss what they see when they fit a diamond pattern inside a square. Quicksilver notes that the “extra spaces” (colored red in Figure 4) look similar to the stair-step pattern that the team worked on previously. But Aznx goes on to talk about the four squares on the outer areas of the square, confusing

Quicksilver. That is, as they each try to work out the details of Aznx's view, they realize that they are not *seeing* things quite the same way.

Quicksilver suggests that Aznx show what he means on the whiteboard, so the ambiguity of his proposal can be resolved. Rather than drawing it himself, Aznx asks Bwang to do a drawing, since Bwang said he could see what Aznx was talking about. Bwang has in the past shown himself to be skilled at making drawings on the whiteboard, while Aznx has not tried to draw much.

Bwang draws a very clear diagram on the whiteboard for the diamond pattern when $N=2$ (see Figure 5). As soon as Bwang completes his drawing, he makes explicit the problem-solving proposal that is implicit in Aznx's way of viewing the problem or the pattern: "We just have to find the whole square and minus the four corners." His drawing has made this process very visible. He drew the diamond pattern with white squares and then filled in a large square that the diamond fits into by adding red squares. The red squares fill in symmetrical spaces in the four corners of the diamond pattern.

The group then discusses the view of the diamond pattern fitting into an enclosing square. They eventually realize that some of their observations are only true for the diamond pattern at a certain stage, like $N=2$.

The screenshot shows a whiteboard interface with several elements:

- Top Left:** A text box containing the formula $(1+N)(N/2 + N) \cdot 2$.
- Top Center:** Handwritten text "3rd step" and "step".
- Top Right:** A small diagram of a diamond shape.
- Middle Left:** A 3D perspective drawing of a diamond-shaped structure made of blocks.
- Middle Center:** A handwritten formula $2(n-1) = 4n(n+1) + (n+1)$.
- Middle Right:** A diagram of a diamond pattern for $N=2$ with red squares in the corners of an enclosing square.
- Bottom Left:** A diagram of a diamond pattern for $N=3$ with red squares in the corners of an enclosing square.
- Bottom Center:** A diagram of a diamond pattern for $N=3$ with red squares in the corners of an enclosing square.
- Bottom Right:** A chat log window showing a conversation between users Aznx, Gerry, Quicksilver, and Bwang. The chat includes messages like "Show what u mean on the whiteboard", "i dont get it", "bwang you show him", "since you get it", "we just have to find the whole square and minus the four corners", "The red areas", "are equivalent or the middle square", "Does that make sense?", "Because what about these", "lemme show you", "There's this original square in the pattern first", "Plats....", "Yeah", "Ok keep going", "no", "it's a shrink down version", "of the pattern", "bwang's right", "this is only looking at a specific size", "yes", "i know", "so do you understand", "it now?", "i think so".

Figure 5. Bwang has drawn the white diamond for $N=2$ with red squares filling in the corners of an enclosing square. Quicksilver is pointing to a diamond pattern for $N=3$, also re-drawn lower on the whiteboard.

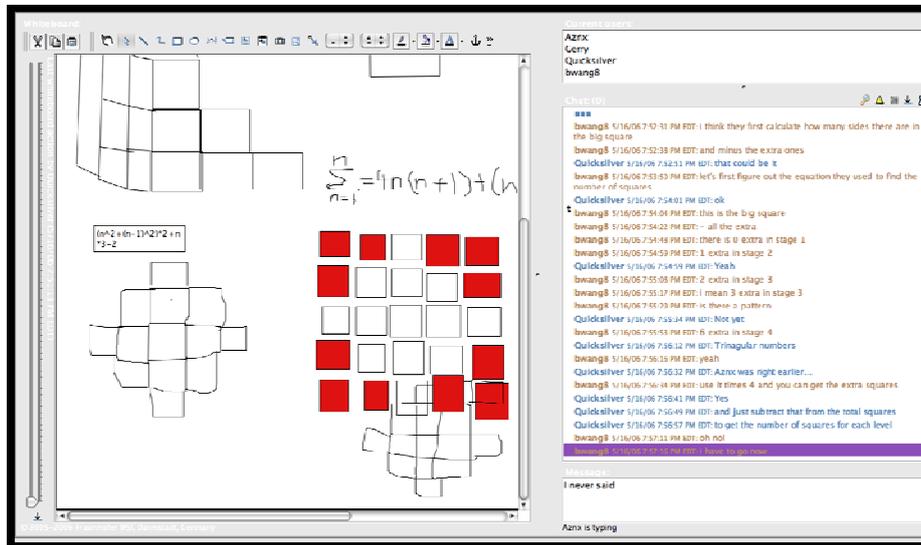


Figure 6. Bwang expanded his drawing to make the diamond for $N=3$. Note the red corners are now stair-step patterns.

So Bwang then draws the pattern for $N=3$. Here it starts to become visible that the red squares in each corner follow the stair-step pattern (see Figure 6).

The group has realized that viewing a graphical image of a mathematical pattern can be very helpful in thinking about the pattern. However, the image captures just one particular stage in the pattern, one value of N . They then start to look at images for different values of N or different stages in the growing pattern. They count the number of red squares in a corner as N increases and notice that it goes: 0, 1, 3, 6. This pattern is familiar to them from their earlier analysis of the stair-step pattern. They call this sequence “triangular numbers,” from Pascal’s triangle, which is often useful in combinatorics math problems. They know that this sequence can be generated by Gauss’ formula for the sum of the consecutive integers from 1 to N : $(N+1)N/2$. Unfortunately, at that point Bwang has to leave the group. But when they return in session 4, they will quickly put together the simple formula for the enclosing square minus this formula for the number of squares in each of the four corners, to solve their problem.

3. Viewing the Learning and Thinking

Let me pause now from all these details about the case study of three students in a virtual math team session and talk about how I view learning and thinking in CSCL groups.

I have tried to demonstrate how I view learning and thinking in CSCL groups by *viewing* with you how a group of three students thought and learned collaboratively within an online environment for drawing and chatting.

Event:	VMT Spring Fest '06, Team B
Session:	session 3 May 16 7 pm
Theme:	"I have an interesting way to look at this problem"
Move:	Show how to view
Pair:	"Can you see how it fits inside a square" "Yes"
Utterance:	"Can you see how it fits inside a square"
Reference:	"it" → diamond pattern

Figure 7. Levels of analysis of online group discourse.

I took you through several levels of analysis of the group discourse (see Figure 7). We started by mentioning the overall context of the *event*. This was an online event in which Team B, consisting of three students, met in the Virtual Math Teams environment to discuss patterns of squares formed by sticks. We then focused on the smaller *session* unit, looking at Team B's third session, in which they considered a pattern that another group, Team C, had analyzed. Within this session, we identified one of several *themes* of discussion in that session, namely the one involving Aznx's "interesting way to look at this problem."

Aznx introduced the theme by initiating a group problem-solving *move*. Namely, he got the group to view the problem in a certain way, as a diamond enclosed in a square. We saw how the group ended up drawing images in their shared whiteboard of diamond patterns enclosed in squares. Aznx introduced this group move in a subtle way; he did not simply come out and say, "We should analyze this pattern as partially filling an enclosing square." Rather, he first announced that he had an interesting view, and then he asked if the others could view the problem in a certain way. He did this through a question/answer response *pair*: he asked a question, which elicited a yes-or-no response from the others. By eliciting the response, he oriented the others to looking at the diagram in the whiteboard in a certain way—namely in the way that he was proposing (Goodwin, 1994; Stahl *et al.*, 2011).

Aznx's formulation of his question looks like a simple *utterance* in question format, but it entails selection from a number of different ways of picturing the relationships among the diamond pattern, the enclosing square, and the empty corners. To begin with, one must decide what the *reference* to "it" is doing.

References like the pronoun "it" are ubiquitous in online text chat. They require the reader to understand or reconstruct the implicit threading or response structure of the chat. The difficulty of doing this often leads to confusions, which require the participants to spend time clarifying the content and structure of their discussion. For instance, in our example of the move of seeing the diamond in the square, the group had to engage in a couple minutes of chatting and drawing to co-construct a shared understanding of the problem.

In other words, in order to view learning and thinking in CSCL groups, I do not try to figure out what is going on in the heads of the students. Rather, I try to figure out what is

going on in their chat postings and their drawing actions. This is what I call the group's *interaction*. In VMT, the interaction of the virtual math team consists of sequences of chat postings and drawing actions.

My first step in figuring out what is going on in the chat postings and drawing actions is generally to try to analyze the sequencing of these by reconstructing their response structure—what previous action each new action is responding to and what kinds of action it is eliciting, what it is opening up an interaction space for, or what kinds of responses it is making relevant as next postings. Often, this leads to some kind of threading diagram (Fuks & Pimentel, 2009), uptake graph (Suthers *et al.*, 2007), or interaction model (Wee & Looi, 2009). This provides a basic structure of the meaning making sequencing. Then I try to understand what problem-solving work is being accomplished at each point in the sequence. This involves looking at different levels of granularity, such as the event, session, theme, move, pair, utterance, and reference. Understanding the meaning that the group is co-constructing in their interaction generally involves going back and forth through these different levels and integrating partial interpretations from the different levels.

Through this process, I can gradually view the learning and thinking that takes place in the CSCL group. This learning and thinking is not something that takes place primarily in the minds of the individual participants (although the individuals in the group are each continuously understanding in some way what is going on and responding to it with their postings and drawings). Rather, when there is an intense collaborative process taking place in the online environment, the thinking and learning takes place in the visible text and graphical interactions.

In a CSCL collaborative interaction, thinking does not take place the way we usually think of thinking. Thought, or cognitive processes, do not take place by neurons connecting and firing in a brain; they take place by text postings and drawings referring to each other and building on each other. We will look more at how this takes place in a minute. Similarly, learning does not take place the way we learned about learning. It is not a change in the amount of knowledge stored in a brain. Rather it is a matter of knowledge artifacts being gradually refined through sequences of text postings and graphical drawings that are interrelated and that explicate each other. The knowledge artifacts may be statements about a problem the group is working on, as viewed from a new perspective that the group has developed. The knowledge artifact might be a drawing like Bwang's in Figure 6 or an algebraic formula that sums up the group's analysis of pattern growth.

4. Unpacking the Group Learning and Thinking

Rather than talking about learning and thinking in the abstract, let us unpack some more how learning and thinking take place in Team B's interaction—in their text chatting and drawing together. Let's go back through the hierarchy of levels of analysis in the opposite order to say something about how references, utterances, response pairs, moves, themes,

Reference:	network of references, indexical ground, joint problem space
Utterance:	recipient design for reading's work
Pair:	projection and uptake
Move:	getting the problem-solving work done
Theme:	coherent interactional sequences
Session:	temporal structuring and re-member-ing
Event:	forming groups and co-constructing knowledge objects

Figure 8. Levels of learning and thinking in online group discourse.

sessions, and events can contribute to learning and thinking in CSCL groups (see Figure 8).

4.1. *Reference: Network of meaning, indexical ground, joint problem space*

When one studies logs of virtual math teams, one sees that they spend a lot of time reaching shared understanding about references in their postings. My conference paper later this week reviews an example of this from Team B's session four, where Aznx, Quicksilver, and Bwang get quite confused about references from the chat to different equations written on the whiteboard (Stahl *et al.*, 2009).

The reason that people devote so much time and energy to resolving confusing references is that the network of references that they build up together plays an extremely important role in their group learning and thinking. In the theory of CSCL, there is considerable emphasis on the ideas of "common ground" (Clark & Brennan, 1991) and "joint problem space" (Teasley & Roschelle, 1993). A group establishes common ground largely by reaching a shared understanding of how references work in their discourse. As it interacts over time, a group co-constructs a network of references that can become quite complex (Sarmiento & Stahl, 2008).

This network of references defines the context or situation in which the group discourse continues to take place. Aznx's reference to "it" that we looked at contributed to a network of meaning that the group built up continuously through their interaction. This network included images of sticks in various patterns (like diamonds at stage N=2 and N=3), the relationships of the patterns (like a diamond enclosed in a square with stair-step empty corners), concepts referred to by technical terms (like "triangular numbers" or "summation"), and symbols representing mathematical operations (like equations for number of squares in a pattern).

As a group builds up its network of shared references, it can use more shortcut references to point to things without creating confusion. People can use deictic references to point to things in the network, like "this formula," "the second equation," or "it." In linguistic terms, the shared network of references provides a background for referring to things, a so-called indexical ground of deictic reference (Hanks, 1992).

In problem-solving terms, the network of references forms a joint problem space, a shared view of the topic that the group is addressing (Sarmiento & Stahl, 2008). For

Team B, the joint problem space starts with the stair-step pattern and the chart of the number of sticks and squares for each stage of this pattern as presented in the topic description for the event. By the middle of session 3, it includes the diamond pattern and the view of “it” enclosed in a square, forming empty corners. It also includes triangular numbers and their associated formula, as well as several other equations from Team C and from Team B’s own work. The team’s interaction (the text postings and drawings) gradually creates this joint problem space and is situated within it. The work and utterances of the team can only be understood (by the participants and by us as researchers) through an on-going understanding of the joint problem space as a network of meaningful reference.

4.2. Utterance: Recipient design for reading’s work

While both the students who participate in the sessions and the researchers who analyze the logs need to understand the network of references, they understand them in very different ways. The students understand how to respond to what is going on the way they might understand how to ride a bike down a hillside. That is, they are not reasoning about it explicitly, rationally, logically, consciously. Rather, they are paying attention to what is going on and responding knowingly and intuitively. Quicksilver has not carried out the kind of analysis of Aznx’s word “it” the way I did; yet he could respond to it with a sophisticated set of questions. He only had a couple of seconds to respond, whereas I could spend hours going back and forth over the log reasoning about explicit interpretations.

People are incredibly skilled at using language without thinking about how they do it. In fact, even researchers are only aware of a small percentage of what people take into account almost instantaneously without being aware of it. We say that Aznx “designs” his announcement and proposal so that it will be read by Bwang and Quicksilver in a way that will lead them to understand in a complex way. They will figure out what “it” is referencing, but also realize some of the ambiguity of the reference. They will also come to think about the strategy for finding the number of squares in the diamond pattern because of this ambiguity. But Aznx does not design his statement explicitly, through a rational sequence of logical arguments. Rather, as a skilled user of language, he gives voice to a well-designed posting that responds to the current discourse situation. It is somewhat like the way a skilled off-road biker responds to the terrain intuitively as she is speeding down a rough hillside with no time to think about what she is doing—and she somehow designs an optimal path for her journey.

Aznx was successful in designing his question so that it would be read in a certain way within the context of the group’s discussion in their joint problem space. This is what ethnomethodology calls the “accountability” of utterances (Garfinkel, 1967). We call this “recipient design.” This simply means that utterances are designed to be understood by their recipients, by the audience for whom they are intended. That is, utterances are designed to meet the expectations of their recipients (Garfinkel, 1967). In

chat, postings are designed to be read in a certain way by the other chat readers. This is in contrast to utterances in spoken talk, which are designed to be heard, and are therefore given subtle vocal emphasis and timing. Chat postings, on the other hand, can incorporate capitalization, abbreviations, symbols, punctuation, emoticons, and special fonts. They can reference previous postings that occurred further back in time because the chat text is persistent, remaining visible or retrievable for longer than speech. In chat, group work takes place as reading; chat postings must be designed to support reading's work of understanding the posted utterances in their discourse context (Livingston, 1995; Zemel & Çakir, 2009).

4.3. Response pair: Projection and uptake

An important aspect of the design of utterances or postings is how they are designed to fit into what comes before and after them. In general, an utterance performs an uptake or response to something that came before (Suthers *et al.*, 2010). At the same time, it elicits a follow-up, or at least makes relevant certain forms of subsequent utterances by others (Schegloff, 2007). Through its uptake and projection, an utterance provides continuity to the discourse—in fact, it thereby creates a temporal structure (Heidegger, 1927/1996).

The clearest and simplest example of this is the adjacency pair or response pair, such as a question/answer pair. A question elicits an answer. That is, stating a question projects that an answer will be given in response. It opens a conversational space for an answer. It makes it relevant for the next utterance to be an answer responding to the question. In other words, a question is designed to be read as something that should be responded to with an answer. A question worded like “Can you see how it fits inside a square?” is designed to be answered with a “yes” or a “no.” The question-and-answer pair forms a unity, a small unit of interaction between people. The “yes” response shows that the posting it is responding to was read as a question and creates the pair as a successful question/answer interaction.

One of my first discoveries in studying virtual math teams was that math discourse is largely driven forward by what I called “math proposal response pairs” (Stahl, 2006a, 2006b). These have the following structure:

- An individual makes a bid for a proposal to the group suggesting how the group should continue to do its mathematical work.
- Another member of the group accepts (or rejects) the proposal on behalf of the group.
- This is the simple, default form of the math proposal response pair. If the proposal is accepted, then work begins on the proposal, often in the form of a follow-up proposal.

Of course, there are many variations and complications possible. The bid can be ignored or never responded to. In that case, it does not function as an effective proposal; at best it is a “failed proposal.” Before a proposal response is made, there can be other response pairs inserted in the middle of the expected pair—such as a clarification question. It is also possible that someone will propose an amendment to the proposal bid

before the original is accepted. Thus, a simple pair can develop a complicated recursive structure of insertions, extensions, repairs, etc.—with each of these being subject to their own insertions, extensions, or repairs. Eventually, each of the intervening pairs may get closed with its anticipated response and then the original pair may be completed.

4.4. Move: Getting the problem-solving work done

Group problem-solving moves often have the structure of a longer sequence than a simple pair. Such a longer sequence may consist of a complex of response pairs embedded in one another. To identify such a structure, it may be necessary to first conduct a threading analysis to determine what is responding primarily to what. Then, it is often useful to see how this longer response sequence is built up out of simple response pairs (Stahl, 2011a).

Together, these intertwining response pairs form a successful move, introducing a new theme for the group. As an example, let's look again at Aznx's move in Figure 9. We can see four response pairs there:

- Aznx announces, "I have an interesting way to look at this problem" and Quicksilver responds by asking him to "Tell us."
- Aznx asks, "Can you see how it fits inside a square?" and first Bwang responds "yes." Then Quicksilver responds, "Yes." Then Bwang responds again, more emphatically, "oh ... yes."
- Quicksilver asks a clarification question about the proposal implicit in Aznx's question, "You are saying the extra spaces ... [?]"
- Aznx, in parallel with Quicksilver's question asks a follow-up question, which contains an implicit further proposal about the group's work: "Also, do you see if you add up the missing areas [...?]"

line	date	start	post	delay		
919	5/16/06	19:35:26	19:35:36	0:00:06	Aznx	I have an interesting way to look at this problem.
920	5/16/06	19:35:41	19:35:42	0:00:03	Quicksilver	Tell us
921	5/16/06	19:35:38	19:35:45	0:00:00	Aznx	Can you see how it fits inside a square?
922	5/16/06	19:35:45	19:35:45	0:00:07	Bwang	yes
	5/16/06	19:35:49	19:35:52	0:00:00	Bwang	[user erased message]
923	5/16/06	19:35:51	19:35:52	0:00:01	Quicksilver	Yes
924	5/16/06	19:35:52	19:35:53	0:00:02	Bwang	oh
925	5/16/06	19:35:55	19:35:55	0:00:06	Bwang	yes
926	5/16/06	19:35:53	19:36:01	0:00:04	Quicksilver	You are saying the extra spaces...
927	5/16/06	19:35:58	19:36:05	0:00:06	Aznx	Also, do you see if you add up the missing areas

Figure 9. The move to introduce Aznx' new way of looking at the group's problem.

As the discussion continues, Quicksilver responds to Aznx's question and the two of them continue to discuss the issues raised in both their questions.

4.5. Theme: Coherent interactional sequence

Aznx's *move* introduces the *theme* of the diagonal pattern viewed as enclosed in a square with missing spaces in the four corners. As we have just seen, the move consists of multiple response pairs that drive the work of the group to consider this theme.

As the theme evolves, the group draws and discusses some increasingly elaborate drawings to view the patterns that the theme involves. The group considers different stages of the pattern ($N=1, 2, 3, 4$) and how the number of missing spaces changes as the diamond pattern grows.

This leads them right to the point where they can formulate an equation to summarize their analysis of the pattern growth. Unfortunately, Bwang has to leave the session and they do not complete this work. During the fourth session two days later, the group picks up this theme and discusses it repeatedly, eventually deriving the equations for number of squares and sticks in the diamond pattern at all stages (Stahl, 2011a). This theme is the basis for the equation for number of squares, which simply subtracts the number of missing spaces in the four corners of a square that encloses the diamond pattern.

4.6. Session: Temporal structuring and re-member-ing

After Bwang left the third session, Aznx and Quicksilver try to review the group's accomplishments. They become confused about various equations and unsure of their ability to explain what the group has figured out. They end the session with Quicksilver saying, "then let's pick it up next time when Bwang can explain it." This ends one session and projects what will happen in a future session.

When the group meets for the fourth session, Aznx and Quicksilver do eventually get Bwang to review the derivation of the equation based on the view of the problem that Aznx introduced in the theme we just considered. The discussion in session four refers back to the group's work in session three and also to Team C's work in session two. But it does this in ways that are situated in Team B's session-four context (Sarmiento-Klapper, 2009). The team members and the memories they bring with them from the past are re-constituted in the new situation, made relevant to the current themes, problem space, and available resources.

4.7. Event: Forming groups and co-constructing knowledge artifacts

At the beginning of session one, the students were not part of a particularly effective group or team. They did not build much on each other's contributions and were hesitant to make proposals, ask each other to undertake tasks, produce permanent drawings, or manipulate mathematical symbols. That all changed dramatically during their four-session event. By the end they had many graphical, narrative, and symbolic

representations or expressions related to their mathematical topic. They worked effectively together and solved their problems well. Problem-solving methods that one person introduced were later proposed and used by the other group members.

You may be wondering if each of the students learned mathematics. The interesting thing about looking closely at what really went on in this event is that what we traditionally consider to be the math content actually plays a relatively minor role in the group's problem-solving. Yes, content is brought in: the students talk about triangular numbers and they apply the formula for summing consecutive integers, for instance. Often, this math content is brought in quickly through proposals by individuals. It is then discussed through responses to the proposal that check that everyone understands the math content and agrees on its applicability. However, the bulk of the hard work is not accessing the traditional math content, but selecting, adapting, integrating, visualizing, sharing, explaining, testing, refining, building on, and summarizing sequences of group response pairs. These proposals and discussions reference not only math content, but also various related resources that the group has co-constructed.

The learning and thinking of the group takes place through the group's discourse, as a temporally unfolding multi-level structure of response pairs interwoven into larger sequences of group moves, problem-solving themes, and sessions of events. The group learns about the mathematics of its topic by building and exploring an increasingly rich joint problem space. It thinks about the mathematical relationships and patterns by following sequences of proposals, raising and responding to various kinds of questions, and engaging in other sorts of interactional moves. Some of this gets summarized in persistent knowledge artifacts like drawings, concepts, equations, solution statements, and textual arguments. The building of the joint problem space generally requires a lot of work to resolve references and to co-construct a shared network of meaning (Stahl *et al.*, 2011).

The math skills—like following certain procedures to do long division or to transform symbols—are not where the deep learning takes place and real knowledge is involved. Rather, the ability to sustain progressive inquiry through methods of group interaction is the real goal. This ability makes use of the math skills as resources for answering questions and coming up with new proposals.

If you wonder how to view learning and thinking in CSCL groups, follow Wittgenstein's advice: "Don't think, look!" My colleagues and I have tried to do this by looking at the work of virtual math teams in the way I have just described. We have been amazed to discover that collaborative learning and group cognition are a lot different than people thought.

5. CSCL as a New Approach to Computers in Education

Reading is learning, but applying is also learning and the more important kind of learning at that ... It is often not a matter of first learning and then doing, but of doing and then learning, for doing is itself learning.

(Zedong Mao)

Computers in education bring many advantages, even as seen within a traditional view of education:

- They give students and teachers access to all the information on the Web.
- They provide the ability to access lectures anywhere/anytime/on large scales.
- They can support testing, tutoring, and scripting of learning processes.
- They offer simulations, educational gaming, virtual reality, and artificial intelligence.

But networked computers in education—using CSCL software environments like VMT—also open opportunities for a radically new view of learning and thinking:

- Networking of students can let them get together with others interested in similar things around the world.
- Effective collaborative learning experiences help students learn how to work, think, and learn in groups. Group work is a new force of production in the world and students need to learn how to produce knowledge in teams.
- CSCL events can give students first-hand, hands-on experience in knowledge building.
- Discussing math in peer groups teaches students how to do math, how to talk math, how to make math connections, how to learn math and think mathematically.

In this second view of computers in education, book learning of facts and rote procedures has a place, but the more important kind of learning comes through doing. CSCL groups can provide effective learning experiences in which teams of students actually do mathematics by exploring rich problem spaces and discussing them the way that Aznx, Quicksilver, and Bwang did.

There are *two* popular approaches to CSCL theory:

- Collaborative learning can be seen as an *extension* of traditional *individual* learning. Individuals possess knowledge that they can state in sentences and can communicate to other individuals. Our commonsense concepts can describe this and we can measure what individuals know at different times. Learning in this traditional view is an increase in individual knowledge.
- Collaborative learning can be viewed as being *qualitatively* different from traditional individual learning, and we need to *discover* the nature of collaborative learning and its relation to individual learning by exploratory research. We need to *re-think* our ideas about learning, collaboration, education, computer support, research methodology, and cognitive theory (Stahl, 2006a). We need to look carefully at data

from real CSCL sessions to see what *actually* takes place there, without imposing our commonsense views.

It should be clear by now that *I view* learning and thinking in CSCL groups as a mystery to be investigated, not as something well understood to be measured. It is a new form of human existence with great potential. We must observe it to learn how it works. My colleagues and I have begun to do this, as have other researchers in CSCL. I have tried to indicate to you here how you can go about observing learning and thinking in CSCL groups.

It may be easier to understand issues of technology design and of traditional instruction when studying computers in education than to understand this new view of learning and thinking. But I believe that if we hope to get the most benefit from computers in education and to understand how groups learn and think in CSCL groups, then we will have to closely observe the discourse and interaction in ways similar to what I have presented here.

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